

Strand II – Number Sense and Operations:

The student demonstrates number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (K – 5): The student understands place value of whole numbers, composes and decomposes whole numbers, understands the operations and their effects on numbers, and solves problems with fluency, using a variety of methods.

Performance Standards:

Kindergarten:

Counting Skills

- **Counts** orally from 1 to 30 and backward from 10 to 1.
- **Reads** numerals to 20 in and out of sequence.
- **Writes** numerals to 10 in order.

Number Meaning

- **Demonstrates** one-to-one correspondence by keeping track of quantities while counting.
- **Creates** a set of a given size.
- **Connects** numerals to the quantities they represent.
- **Demonstrates** that the meaning of a number does not change no matter how objects are grouped.

Number Relationships

- **Combines and separates** sets of objects with quantities and **identifies** the parts and the whole.
- **Orders** quantities from least to most or most to least.
- **Describes** a quantity using the terms more, less, and same.
- **Estimates** quantities and **explains** reasoning.

Number Operations

- **Represents** numbers in a variety of equivalent ways (e.g., dots, pictures, numerals).
- **Combines and separates** two single-digit numbers using pictures, stories, and objects to model the situation.
- **Records** numerical information using pictures, words, and/or numbers.

Fractions

- **Explores** halves through part-whole relationships using real objects (e.g., cut several apples into 2 equal parts).

First Grade:

Counting Skills

- **Counts** orally to 100.
- **Reads, writes, and sequences** numbers to 100.
- **Counts** orally by 2s to 30 and by 5s and 10s to 100.

Number Meaning

- **Forms and counts** groups of objects (e.g., 2s, 5s, 10s).
- **Groups** objects by 10s and 1s to explore place value (e.g., 2 tens and 4 ones = 24).

Number Relationships

- **Represents** equivalent forms of the same number (i.e., part – part – whole) through the use of physical models, diagrams, and number expressions to 20 (e.g., $3+5=8$, $7+1=8$, $2+6=8$).
- **Builds, estimates, and compares** combinations of numbers to 20 in different ways using pictures, stories, and objects to model the combinations.
- **Finds** the sum or difference of two quantities up to 20 using symbols in problem-solving situations (e.g., $12+5=17$, $12-5=7$).
- **Examines** the concept of ordered pairs by linking commonly paired relationships (e.g., How many legs do 3 chickens have?).
- **Demonstrates** that the number of objects (up to 20) does not change when the objects are moved or rearranged.
- **Identifies** one more than and one less than a given number.
- **Compares** two three-digit numbers using greater than, less than, and equal to.

Operations

- **Constructs** a mental image of combining and separating quantities in problem situations.
- **Develops** strategies and estimation skills for solving addition and subtraction problems.
- **Records** strategies for solving, combining, and separating problems using pictures, numbers, equations, and words.
- **Identifies** equal shares in examples of models partitioned with equal and unequal parts (e.g., share 12 M&M candies among 3 friends, share 11 apples among 5 friends).
- **Uses** estimation and other methods to justify whether or not an answer is reasonable (e.g., Will 25 marbles fit in this small container?).

Fractions

- **Examines** halves and fourths using real objects or manipulatives.

Second Grade:

Counting Skills

- **Reads, writes and sequences** numbers to 200.
- **Explains** numbers to 1,000 using a variety of strategies (e.g., manipulatives, numbers, drawing, words).
- **Counts** by 2s to 100, by 5s and 10s to 200 and by 100s to 1,000.

Number Meaning

- **Applies** patterns in skip counting; **compares and defends** the relationship between skip counting, grouping, and equal sets.
- **Explains** that digits stand for different amounts in different place value positions (e.g., 22 means 2 groups of ten and 2 ones).

Number Relationships

- **Develops** fluency with 10 as an important (anchor, landmark, friendly) number and **applies** it in problem situations.
- **Analyzes** relationships of quantities to 20 using part – part - whole (e.g., $1+7$, $2+6$, $3+5$, $4+4$: all equal 8.).
- **Uses and explains** strategies for recalling addition and subtraction facts to 18.
- **Makes and defends** a reasonable estimate of quantities up to 200 in problem situations.
- **Identifies** number sequences (e.g., 12, 14, 16... what comes next?).

Number Operations

- **Uses and explains** multiple strategies to solve addition and subtraction problems, with and without re-grouping, using two-digit numbers.
- **Decomposes and recombines** numbers in logical ways to solve problems (e.g., $8+5 = (3+5)+5 = 3+(5+5) = 3+10=13$).
- **Writes** addition and subtraction equations for problem situations with one- or two-digit numbers using vertical or horizontal formats.
- **Develops** story problems to match given equations.
- **Uses and explains** mental math strategies to solve problems (e.g., children solve problems without pencil and paper).
- **Solves** the two types of situational subtraction problems: comparing two sets of objects and separating parts from whole.
- **Describes** the inverse relationship between addition and subtraction.
- **Estimates** sums and differences of one- and two-digit numbers and **explains** whether the answer is reasonable.
- **Explains** properties of addition (e.g., associative, commutative, and identity property of 0).

Fractions

- **Models** how many parts make a whole using equal fractional parts.

Third Grade:

Place Value

- **Reads, writes, models and orders** numbers to 1,000.
- **Uses** models to demonstrate equivalent representations of any two-digit number (e.g., $22=20+2$; $22= 10+0+2$; $22= 10+ 12$).
- **Estimates** quantities up to 1,000 in a variety of problem-solving situations and **explains** the strategy used (e.g., anchor numbers).
- **Explains** the purpose of 0 on a number line.

Number Relationship

- **Explains** the relationship between skip counting and multiplication.
- **Skip counts** by important single-digit numbers (e.g., 2, 3, 5), two-digit anchor numbers (e.g., 10, 20, 25), and odd/even sequences.
- **Explains** the relationship among commonly encountered factors and multiples (e.g., factor pairs of 12: 1×12 , 2×6 , 3×4 ; multiples of 12: 12, 24, 36...).
- **Identifies** factors of 100, **explains** their relationship to 100, and **uses** them to understand the structure of 1,000.

Addition and Subtraction

- **Explains** strategies used to automatically recall basic addition and subtraction facts.
- **Selects and uses** an appropriate method for adding and subtracting using mental arithmetic.
- **Uses and explains** standard addition and subtraction notation (i.e., equations) to represent word problems.
- **Uses and explains** anchor numbers, decomposing, and recombining strategies to combine and compare quantities in the 100s [e.g., $37+54=(30+7)+(50+3+1)=30+50+10+1 =90+1 =91$].

- **Models** concepts of addition and subtraction of two- and three-digit numbers, with and without regrouping, in a variety of ways.
- **Uses and explains** estimation strategies used to solve situational story problems.
- **Solves** addition/subtraction problems with a variety of givens and unknowns.

Multiplication and Division

- **Models, represents, and explains** multiplication and division equations and situations using words, pictures, manipulatives, etc.
- **Explains** the relationship between multiplication and division.
- **Understands** the effects of multiplying and dividing whole numbers (e.g. multiplying enlarges a whole number, dividing decreases a whole number).
- **Uses and explains** strategies for recalling multiplication facts to 81.
- **Models** the 2 types of division (sharing and partitioning) based on pattern situations.
- **Uses and explains** multiplication properties (e.g., identity property of 1, associative, commutative, and property of 0).

Fractions

- **Explains** that equal parts or fair shares must be the same.
- **Reads, writes, and uses** conventional fraction words and notation and **links** them to their pictorial representations.
- **Explains** that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.
- **Adds** and **subtracts** fractions that have the same denominator using manipulatives.
- **Explains** common equivalents, especially relationships among wholes, halves, fourths, and eighths, as well as wholes, thirds and sixths.
- **Orders** simple fractions and **identifies** the greater fraction.
- **Uses** fractions to solve everyday problem situations.

Decimals

- **Explores** the theory that fractions and decimals are two different representations of the same concept (e.g., .50 is $\frac{1}{2}$ of a dollar).

Fourth Grade:

Place Value

- **Reads, writes, and orders** numbers to 10,000.
- **Demonstrates** an understanding of place value by modeling and representing the value of digits in a number up to 5 digits.
- **Estimates** quantities up to 10,000 in a variety of situations and **explains the** strategy used.

Number Relationships

- **Finds and counts** by factors of 100 and 1,000 and **uses** them to explain the structure of 10,000.
- **Identifies** all the factor pairs of common numbers (e.g., 12, 20, 24, 36, etc.), and **uses** them to model, and **solves** real-life problems using concrete representations.
- **Explains** why numbers such as 2, 3, 5, 7, and 11 are prime numbers.

Addition and Subtraction

- **Estimates and solves** addition and subtraction problems using a variety of strategies (e.g., mental math, calculators, spreadsheets, anchor numbers, and pencil/paper).

Multiplication and Division

- **Uses and explains** strategies to recall basic multiplication and division facts through 100.
- **Estimates and solves** multiplication/division problems involving multi-digit numbers multiplied/divided by one-digit numbers using a variety of efficient strategies [e.g., mental math, anchor numbers, distributive property (decomposing and recombining)] and **determines** if the answer is reasonable.
- **Uses and explains** how division notation represents the division situations of sharing and partitioning and why zero cannot be a divisor.
- **Explains** that what to do with "remainders" in division depends on the situation (e.g., How many cars do you need to take 9 people to the zoo, if each car can hold 4 people?).
- **Explains** the inverse relationship between multiplication and division.
- **Creates, writes, and solves** situational story problems involving addition, subtraction, multiplication, or division.
- **Solves** multiplication and division problem situations with a variety of givens and unknowns (e.g., $4 \times 3 = ?$, $3 \times ? = 12$).

Fractions

- **Explains** that equal fractions of a whole have the same area but are not necessarily congruent.
- **Explains** how the "size of the whole" affects the size of the fraction (e.g., Is $\frac{1}{2}$ of \$1.00 better than $\frac{1}{4}$ of \$100.00?).
- **Explains** the meaning of the numerator and denominator in fractional notation.
- **Identifies and constructs** models to represent equivalent fractions, mixed numbers, and improper fractions.
- **Finds** common denominators using concrete representations (e.g., use fraction bars to find the common denominator for $\frac{1}{2}$ and $\frac{1}{3}$).
- **Adds and subtracts** fractions with common and uncommon denominators using a variety of strategies (e.g., manipulatives, numbers, and pictures).
- **Compares** proper, improper, and mixed fractions to fractions and whole numbers using a variety of strategies (e.g., manipulatives, numbers, pictures, and a number line).
- **Explains** the relationship between fractional notation and the concept of division (e.g., $\frac{3}{5}$ means 3 divided by 5; $3 \div 5$).
- **Uses** fractions to solve everyday problem situations.

Decimals

- **Reads, writes, compares, and models** tenths and hundredths in decimal and fractional notation.
- **Represents** decimals and their relationship to fractional equivalents using a variety of strategies.

Fifth Grade:

Place Value

- **Reads, writes, orders, and compares** numbers to 1,000,000.
- **Estimates** quantities up to 1,000,000 in a variety of situations, and **explains** the strategies used.

Number Relationships

- **Uses** a variety of strategies, including calculators and geometric models, to find factors, multiples, primes, even/odd numbers, and square numbers, and to explain number composition.
- **Uses** factor relationships to 100 and 1,000 to solve real-life applications, and **explains** the structure of 1,000,000.

Addition, Subtraction, Multiplication, and Division

- **Develops, compares, and selects** a strategy that is efficient and accurate when solving addition, subtraction, multiplication, and division problems.
- **Selects** the appropriate operation involving addition, subtraction, multiplication, and division from situational story problems, and **uses** relationships among the four basic operations to solve them.
- **Estimates** and **solves** problems involving sums, differences, products, and quotients and **justifies** the reasonableness of the solutions.

Fractions

- **Finds** common denominators using concrete representations and other strategies (e.g., uses fraction bars to find the common denominator for $\frac{1}{2}$ and $\frac{1}{3}$).
- **Uses** addition and subtraction of mixed numbers in problem solving situations.
- **Uses** fractions and decimals to help solve everyday problem.
- **Estimates** and solves problems involving addition and subtraction of fractions, and **justifies** the reasonableness of the solution.
- **Describes** the relationship between repeated addition of fractions and multiplication of fractions by whole numbers (e.g., $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4} = \frac{3}{4}$).

Decimals

- **Compares and orders** decimals to other decimals and decimals to whole numbers using models and notation (e.g., $>$, $<$, $=$).
- **Explains** the connection between fractions, decimals, and percents; **applies** appropriate models, notations, and reasoning to represent these relationships.

Strand II – Number Sense and Operations:

The student demonstrates number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student understands problems involving fractions, decimals, and percents and develops, analyzes, and explains a variety of *algorithms* and methods to solve problems.

Performance Standards:

Sixth Grade:

Simple Expressions

- **Explains** the effect of changing the placement of mathematical symbols on the outcome of the problem (e.g., \$1.00, \$10.00, \$100.00).
- **Uses** commutative, associative, identity, zero, and distributive properties when solving problems.

- **Explains and demonstrates** that order of operations and properties apply consistently across all math topics.
- **Selects** an appropriate operation (i.e., +, -, x, ÷) to solve situational story problems.

Rational Numbers/Fractions

- **Selects and uses** the appropriate number form (e.g., fraction, decimal, or percent) in a variety of situations, including measurement in U.S and metric systems.
- **Explains** the part-whole relationships in division situations (e.g., $\frac{1}{2} = 1 \div 2$).
- **Orders** a mix of fractions, decimals, and percents.
- **Describes** patterns within and among sets of fractions, decimals, and percents (e.g., if $\frac{1}{8} = .125$, $\frac{2}{8} = .250$, what does $\frac{7}{8}=?$).
- **Describes** the effects of arithmetic operations on fractions and decimals.
- **Recognizes and uses** prime and composite numbers.
- **Finds** Greatest Common Factor (GCF) and Least Common Multiple (LCM) using a variety of strategies, including prime factorization.
- **Develops and tests** strategies for adding and subtracting fractions with like and unlike denominators.
- **Develops and tests** strategies for multiplying and dividing proper, improper, and mixed fractions.
- **Translates** hypotheses into formal and fluent fractional and decimal computations using appropriate mathematical terminology and processes.
- **Estimates and solves** problems involving fractions and **justifies** the reasonableness of the solution.

Decimals

- **Develops and tests** strategies for adding, subtracting, multiplying, and dividing decimals.
- **Estimates and solves** problems involving decimals and **justifies** the reasonableness of the solution.

Estimation

- **Uses** the appropriate estimation strategy for a variety of situations.
- **Determines** when an exact answer is necessary or when an estimate is appropriate (e.g., medicine dosage vs. number of people at a concert).

Seventh Grade:

Integers

- **Compares and orders** integers (positive and negative whole numbers).
- **Identifies and applies** commutative, associative, zero, distributive and identity properties consistently.
- **Develops and tests** strategies for adding, subtracting, multiplying, and dividing integers.
- **Describes** the effects of arithmetic operations with integers
- (e.g., $+ 8 + - 3 = 5$).
- **Translates** problem-solving strategies into efficient computation using appropriate mathematical terminology.

Exponents and Square Roots

- **Explains and models** the value of exponents and square roots.
- **Simplifies and evaluates** (solves) numerical expressions involving exponents (e.g., $2^3 = 2 \times 2 \times 2 = 8$).

- **Develops and tests** strategies for multiplying and dividing like-based exponents (e.g. $2^3 \times 2^5 = 2^8$).

Scientific Notation

- **Uses** the powers of 10 when describing and representing numbers in scientific notation (e.g., $12.50 = 1.25 \times 10^3$).

Proportional Reasoning

- **Explains** the relationship that can be expressed as ratios of part-to-whole (e.g., 5 red apples out of a total of 8 apples, expressed as $5/8$).
- **Explains** the relationship that can be expressed as part-to-part (e.g., 5 red apples, 3 green apples, expressed as $5/3$).
- **Explains** relationships that can be expressed as proportions or percents (e.g., $1/2 = 50\%$).
- **Develops** meaning for percents greater than 100 and less than one (e.g., It is possible to have 120% or $1/2\%$).
- **Develops** more than one strategy to solve real-life problem situations involving ratios, proportions, and percents.
- **Estimates and solves** problems involving proportions and **justifies** the reasonableness of the solution.
- **Creates and writes** story problems involving proportions.

Eighth Grade:

Rational and Irrational Numbers

- **Categorizes** numbers and sets within structures of the real number system (e.g., Natural numbers < rational numbers < real numbers).
- **Shows** flexibility using multiple number representations; **identifies** relationships involving the subsets of the real number system (e.g., order, least to greatest: 1, $\sqrt{2}$, $\sqrt{3}$, 2).
- **Selects** appropriate mathematical representations to describe thought-provoking real-life situations.
- **Manipulates** all real numbers, their properties, and operations.

Exponents

- **Simplifies and evaluates**, if solvable, algebraic expressions for all types of real numbers including exponents and common square roots.
- **Examines, describes, and models** exponential patterns that reflect growth and decay (e.g., Represent doubling 1¢ every day for 10 days in exponential form).

Number Theory

- **Develops and evaluates** arguments involving real numbers, their patterns and operations.
- **Develops and uses** strategies to estimate the results of rational-number computations and **judges** the reasonableness of the results.