Teacher Instructions: Perplexing Palindrome Problem

Grade Level: K - 2

Task: Perplexing Palindrome Problem

Standard: Patterns, Functions, and Algebraic Concepts

A palindrome is something that can be read in the same way frontwards and backwards. Palindromes can be words or numbers. An example of a palindrome is: dad

Two friends create a palindrome game using one number cube. They want to find all the 3-digit palindromes that can be created using digits on a number cube.
How many different palindromes could they make?
Remember to show all your work!

Context – From the Task Author: This problem was given to a group of first and second graders who had been working on place value. The students that attempted this problem had worked with numbers and had played many different place value games, such as BAGEL, PICO, and FERME.

What the task accomplishes…

- The task provides the opportunity for students to use their knowledge of place value to create various numbers that can be read the same frontwards and backwards. Of course, one does not really have to have a true understanding of place value in order to complete this problem.
- The task could measure place value skills by requiring students to write out the number they created in expanded notation.

What students will do…

- Most of the students began by creating a list of numbers.
- Nearly all started with a one in the hundreds column and worked through the numbers in order.
- This was great to see as it showed that the students had a handle on systematic lists that would ensure they had all combinations.

Time Required: Less than 1 hour.

Interdisciplinary Links: A language arts link is the most obvious one. Students could use letters to create palindromes.

Teaching Tips…

- To make this problem more challenging, students may be required to find all the possibilities of 4 and/or 5 digit palindromes using the number cube. Students can then be asked to compare the results and make mathematically relevant observations.

Kindergarten Modifications: This performance task is predominately for first and second graders, but can be easily modified for Kindergarten and beginning first grade students. Let the students identify palindromes by manipulating number tiles using numbers 1 – 5. Allow students to make as many palindromes as they can, or give them a goal of 10 to identify using the number tiles.

Suggested Materials: Number cubes and/or number tiles so that students can manipulate the numbers as they work.

Possible Solution…

- ✓ 111, 121, 131, 141, 151, 161, 212, 222, 232, 242, 252, 262, 313, 323 333, 343, 353, 363, 414, 424, 434, 444, 454, 464, 515, 525, 535, 545, 555, 565, 616, 626, 636, 646, 656, 666.
- ✓ 36 palindromes can be made. Students may notice that the hundreds digit and the ones digit must be identical in order to create a palindrome.
- ✓ There are 6 digits on the number cube which can be used for the hundreds and ones places, and 6 digits on the number cube that can be used for the tens place. 6 x 6 is 36 possible palindromes.

Benchmark Descriptors:

- ❑ The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- ❑ The descriptors are generalizations of what student work could look like.
- ❑ It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- ❑ It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Task Specific Rubrics: The intention of the *Performance-Based Mathematics Task Bank* is to provide teachers with materials and experience in working with formative assessments. Teachers are encouraged to modify tasks and/or the task's rubrics to meet the individual needs of their class.

- ➢ Each monthly task comes with a task specific rubric designed around the tasks grade band K – 2, 3 – 5, or 6 – 8. The task specific rubric may not be appropriate for scoring student work at all of the task's intended grade levels because the scope of performances for each rubric covers 3 grade levels. The rubric may not meet the teacher's specific need for administering the task. Teachers should modify the rubric based on their intention for using the task. A template for creating a task specific rubric has been included in the task bank.
- ➢ Each teacher should use the task specific rubric, the benchmark descriptors and papers, and the *APS Mathematical Standards* as a guide for assessing their student's performance on a task.

Novice	✓ The novice will show a beginning level of understanding of the problem. ✓ S/he may begin by listing various numbers in columns that may or may not be palindromes. ✓ The novice may not use all digits 1-6 on the number cube. ✓ The novice will use little or no math language, and will provide no explanation or evidence of what was done to solve the problem.
Apprentice	✓ The apprentice may show a list of numbers that fit the palindrome requirements, showing some understanding of what the problem was asking. ✓ S/he may not find all palindrome combinations. ✓ S/he may not create a detailed or labeled representation, but may use some math language and words/text to explain her/his thinking and reasoning. ✓ There may be several gaps in the student's reasoning.
Practitioner	✓ The practitioner will clearly understand the problem and will create a math representation to show all of the palindromes that could be made. ✓ The work will be correct and there will be an accompanying explanation that shows how S/he went about this problem. ✓ Work will be labeled, and correct math language will be used.
Expert	✓ The expert will show a deep understanding of the problem. These students will show a logical and efficient approach to solving the problem by finding all correct palindromes that follow a pattern. ✓ Students will make mathematically relevant observations such as you can “count up by 10 for each group". ✓ These students may also take the task a step further by extending the problem to palindromes with more than 3 digits, or commenting on the relative value of the palindromes created.

APS Mathematical Standards…

❖ **The math standards stated for this task are aligned to the APS Draft Standards 2000.**

Strand – Patterns, Functions and Algebraic Concepts:

The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focuses on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (K – 5): The student sorts and classifies objects by properties, extends, analyzes, and generalizes patterns, and models the result using appropriate tools. The student understands qualitative and quantitative change and begins to use number properties.

Performance Standards:

Kindergarten:

- **Identifies, describes,** and **extends** patterns with familiar objects in both classroom and real-life situations.
- **Creates describes,** and **extends** patterns.

First Grade:

- **Identifies, describes, creates,** and **extends** patterns observed in familiar objects in real-life situations (1 child = 2 eyes, 2 children = 4 eyes).

- **Describes** how a pattern develops, repeats, and builds toward more complex patterns.

Second Grade:
- **Describes, creates** and **extends** a wide variety of patterns.
- **Identifies** patterns in a number system (5, 10, 15…).
- **Develops** and **applies** more complex patterns and relationships in real-life and mathematical problem situations.

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (K – 5): The student will understand place value of whole numbers, compose and decompose whole numbers, understand the operations and their effects on numbers and solve problems with fluency and a variety of methods.

Performance Standards:

Kindergarten:
- **Reads** numerals to 20 in and out of sequence.

First Grade:
- **Reads, writes,** and **sequences** numbers to 100.
- **Groups** objects by 10's and 1's to explore place value (2 tens and 4 ones = 24).

Second Grade:
- **Reads, writes** and **sequences** numbers to 200.
- **Explains** numbers to 1,000 using a variety of strategies (manipulatives, numbers, drawing, and words).
- **Explains** that digits stand for different amounts in different place value positions. (22 means 2 groups of ten and 2 ones).
- **Identifies** number sequences (12, 14, 16… What comes next?).

Strand - Data Analysis, Statistics, and Probability:

The student identifies patterns and special features of data and events of chance through experiences with meaningful mathematical problems that focus on comparing, predicting, representing data, and making decisions to communicate mathematical understanding.

Benchmark (K – 5): the student designs a data question and collects, represents and analyzes data. The student compares representations and understands measures of center. The student predicts outcomes and use likely and unlikely to describe the probability of a given situation.

Performance Standards:

Kindergarten:
- Describes patterns in nature and daily routines.

First Grade:

- Describes regularly occurring patterns in nature and in daily routines.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

Performance Standards:

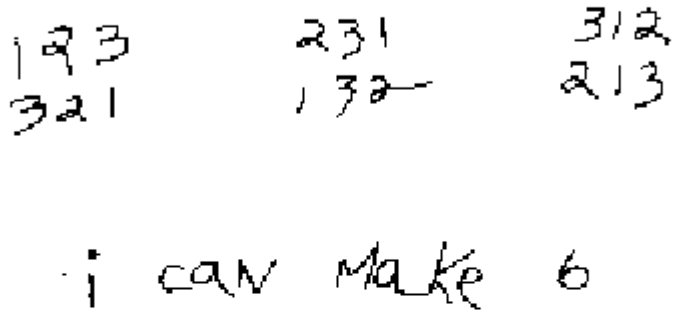
Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies,** and **translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

Novice

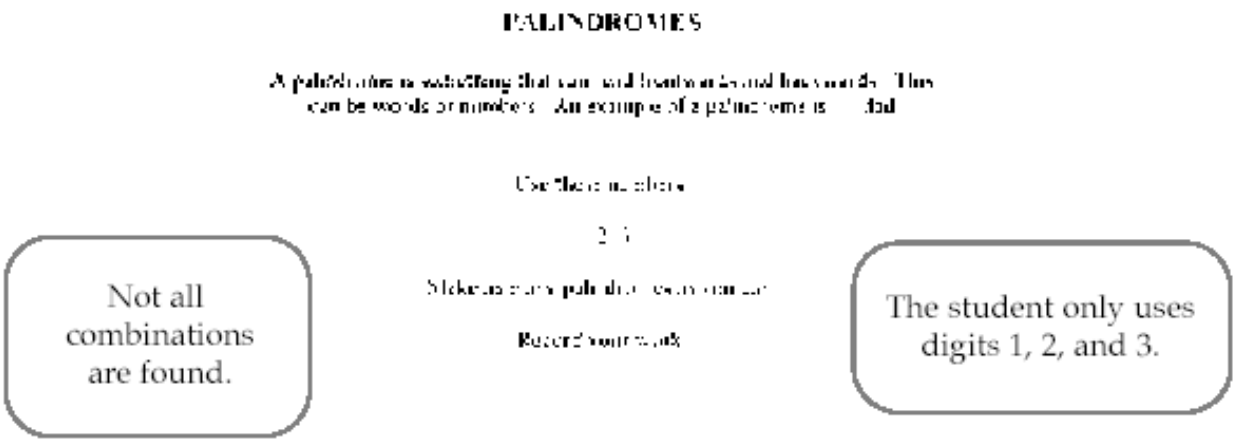
Some reasoning is evident, but that reasoning is incorrect.

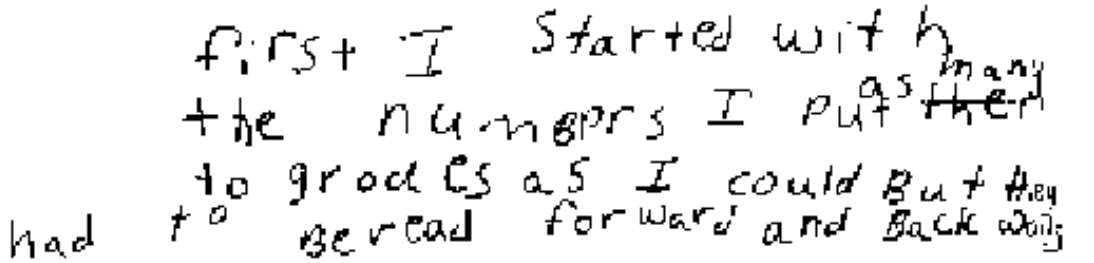


Little or no math language is used.

The student has created a 3-digit number pattern but it is not a palindrome pattern (ABA).

Apprentice



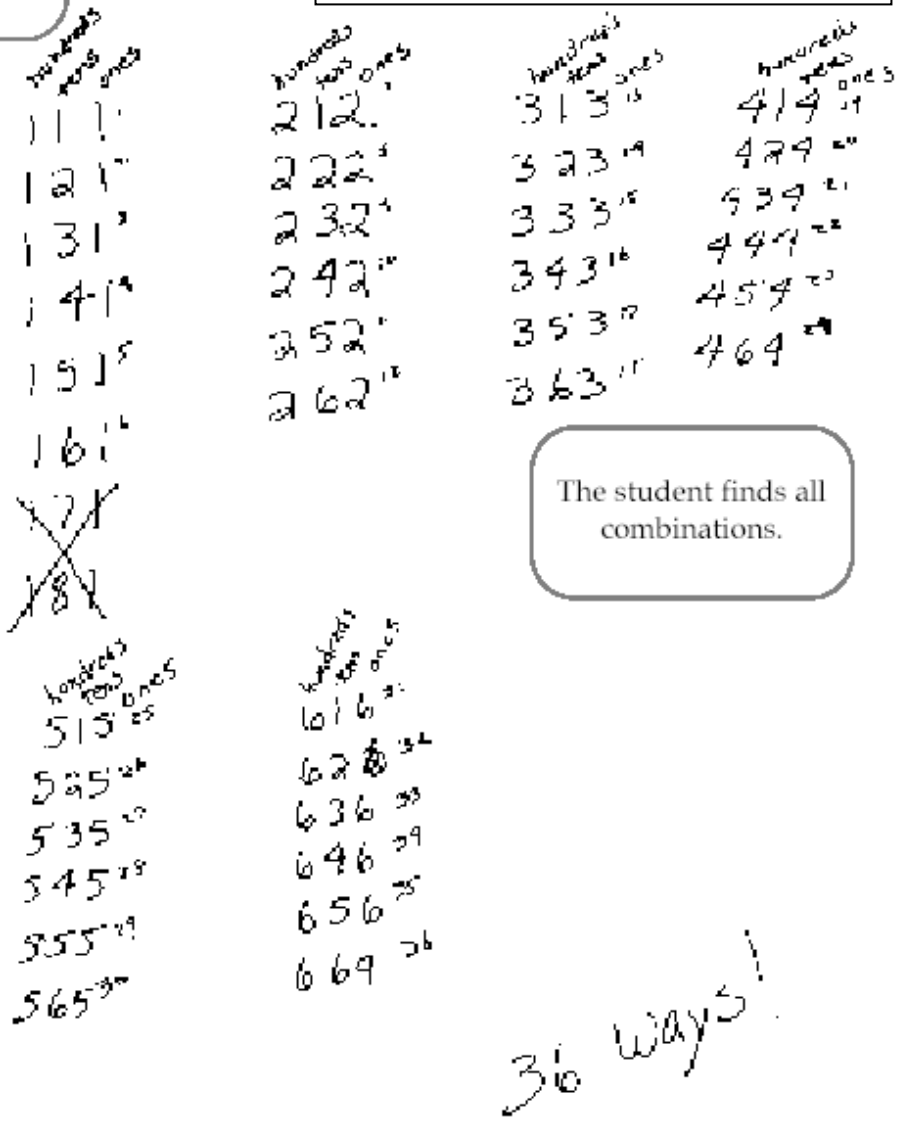


The student demonstrates a beginning understanding of the palindrome number pattern ABA, but cannot identify all of the 3-digit palindrome numbers.

Practitioner

The student has demonstrated an understanding of place value to the hundreds position, and uses it to establish an efficient system for tracking their palindromes.

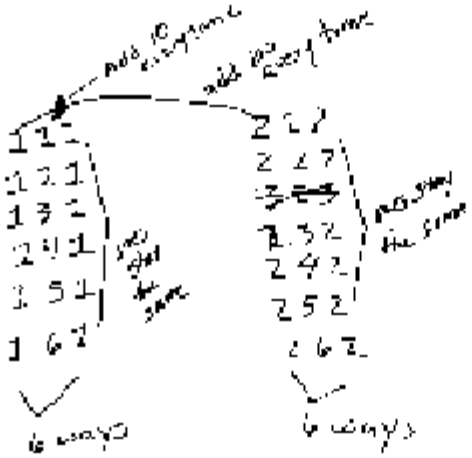
The student uses some math language.



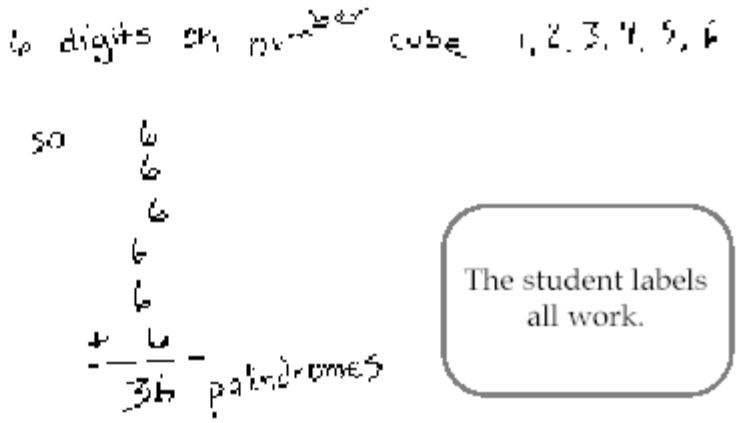
The student finds all combinations.

All work is labeled.

Expert

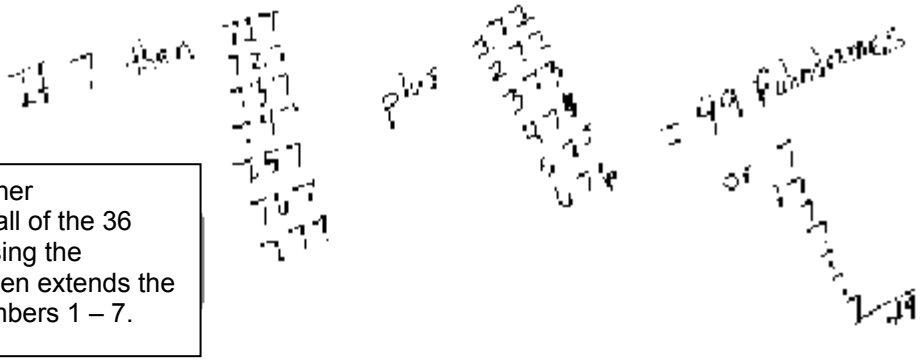


The student makes mathematically relevant observations.



Accurate math language is used.

The student labels all work.



The student uses his/her generalization to find all of the 36 3-digit palindromes using the numbers 1 – 6, and then extends the solution using the numbers 1 – 7.