

# Teacher Instructions: The Height Dilemma

**Grade Level:** 6 - 8

**Task:** The Height Dilemma

**Standard:** Number Sense and Operations

I just found this picture of my parents. Cute, aren't they? I remember my Dad saying that he is 6'2" tall, but I can't remember how tall my Mom is! Mathematically determine how tall my Mom is. Convince me that your strategy and reasoning are mathematically sound. Make any connections and/or generalizations that you can.



**Context – From the Task Author:** My sixth grade class was studying ratios and proportions. I wanted to assess the class to see which students could identify a real-world proportion problem. Applying this concept has been difficult for my sixth graders in the past.

## **What the task accomplishes...**

- This task tells me which students have a sound understanding of proportion.
- It tells me which students are more comfortable with the less formal idea of scale.
- This task also makes students deal with concepts of measurement.

## **What students will do...**

- Some students noticed that my father wasn't standing up straight and thought of how to compensate for that.
- They also noticed that he had a hat on and my mom had puffy hair.
- They also had to decide from where to measure - my dad's toes or the heel of his foot?
- Most students used some kind of division and came out with a fraction or decimal of a foot.
- Many students immediately wrote the remainder or decimal as inches instead of part of a foot, which provided me with useful diagnostic information for future instruction.

**Time Required:** About 45 minutes

**Interdisciplinary Links:** This task could be used as a discussion of scale introduced in a Social Studies unit.

### Teaching Tips...

- Let the students ask questions about the picture before they attempt the problem.
- If possible, allow students in the class to answer the questions.
- When you are giving the task to your students you could substitute a picture of people with whom your students may be more interested, such as your parents, or even you and your spouse, partner, team teacher, colleague or friend!
- **Teacher Note:** The original picture for this task prints dark, please feel free to use a substitute picture, which copies more clearly.

**Suggested Materials:** My students asked for string to measure my parents' heights, rulers (both centimeter and inch), and graph paper.

### Possible Solution...

- ✓ I accepted a range of solutions depending on the accuracy of students' measurements and strategies.
- ✓ My mother is less than 5 feet tall - about 4'10" or 4'11".

### Benchmark Descriptors:

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- The descriptors are generalizations of what student work could look like.
- It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

#### Novice

- ✓ A novice would not be able to find a strategy that would work.
- ✓ They might begin by measuring the height of my dad and my mom in the picture, but then not really know where to go next.
- ✓ They might guess the height of my mother with no mathematical basis.

#### Apprentice

- ✓ An apprentice would most likely use scale (proportion is a pretty sophisticated concept) to find the height of my mom.
- ✓ The apprentice, however, will not be able to complete the scale strategy successfully.
- ✓ It may not be clear how they came up with the scale they use to find my mother's height, or there may be a lot of assuming or guessing.

#### Practitioner

- ✓ The practitioner will use either the scale or the proportion strategy successfully.
- ✓ They will either have an explanation or equations that allow you to follow their strategy.
- ✓ They will use effective math language to communicate, and will show results of their computation.

### Expert

- ✓ An expert will most likely be able to connect this problem to the concept of proportion and will solve the problem more than one way to verify their solution.
- ✓ The expert will use efficient and sophisticated math language and notation, and may make mathematically relevant observations.

### APS Mathematical Standards...

❖ The math standards stated for this task are aligned to the APS Draft Standards 2000.

#### Strand – Patterns, Functions, and Algebraic Concepts:

Students will demonstrate an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

**Benchmark (6 – 8):** The student will use tables, graphs, and symbolic representations of patterns. The student will understand and use variable and linear equations in algebraic problem solving.

#### Performance Standards:

Sixth Grade:

- **Solve** one-step equations using the concept of balance when quantities are added, subtracted, or divided by both sides of an equation.

Seventh Grade:

- **Identify and use** variable expressions and formulas to solve a variety of real-life situations.
- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Develop and test** strategies for solving two-step equations.
- **Translate** hypotheses into formal methods of solving algebraic equations.

Eighth Grade:

- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Identify and model** real-life situations using multiple representations.
- **Develop and test** strategies for solving multi-step equations.
- **Solve** equations for specified variables.

#### Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

**Benchmark (6 – 8):** The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

## Performance Standards:

Sixth Grade:

- **Select** an appropriate operation to solve situational story problems.
- **Select and use** the appropriate number form (fraction, decimal, or percent) in a variety of situations, including measurement in U.S. and metric systems.
- **Develop and test** strategies for adding and subtracting fractions with like and unlike denominators.
- **Develop and test** strategies for multiplying and dividing fractions.
- **Develop and test** strategies for adding and subtracting decimals.
- **Develop and test** strategies for multiplying and dividing decimals.
- **Estimate and solve** problems involving fractions & decimals, and justify the reasonableness of the solution.
- **Use** the appropriate estimation strategy for a variety of situations.
- **Determine** when an exact answer is necessary or when an estimate is appropriate.

Seventh Grade:

- **Translate** problem-solving strategies into efficient computation using appropriate mathematical terminology.
- **Develop** more than one strategy to solve real-life problem situations involving ratios, proportions, and percents.
- **Estimate and solve** problems involving proportions and **justify** the reasonableness of the solution.
- **Create and write** story problems involving proportions.

Eighth Grade:

- **Select** the appropriate representations to describe thought provoking real-life situations.
- **Develop and evaluate** arguments involving real numbers, their patterns and operations.
- **Develop and use** strategies to estimate the results of rational-number computations and **judge** the reasonableness of the results.

## Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

**Benchmark (K - 12):** The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

## Performance Standards:

Grades Kindergarten through twelve:

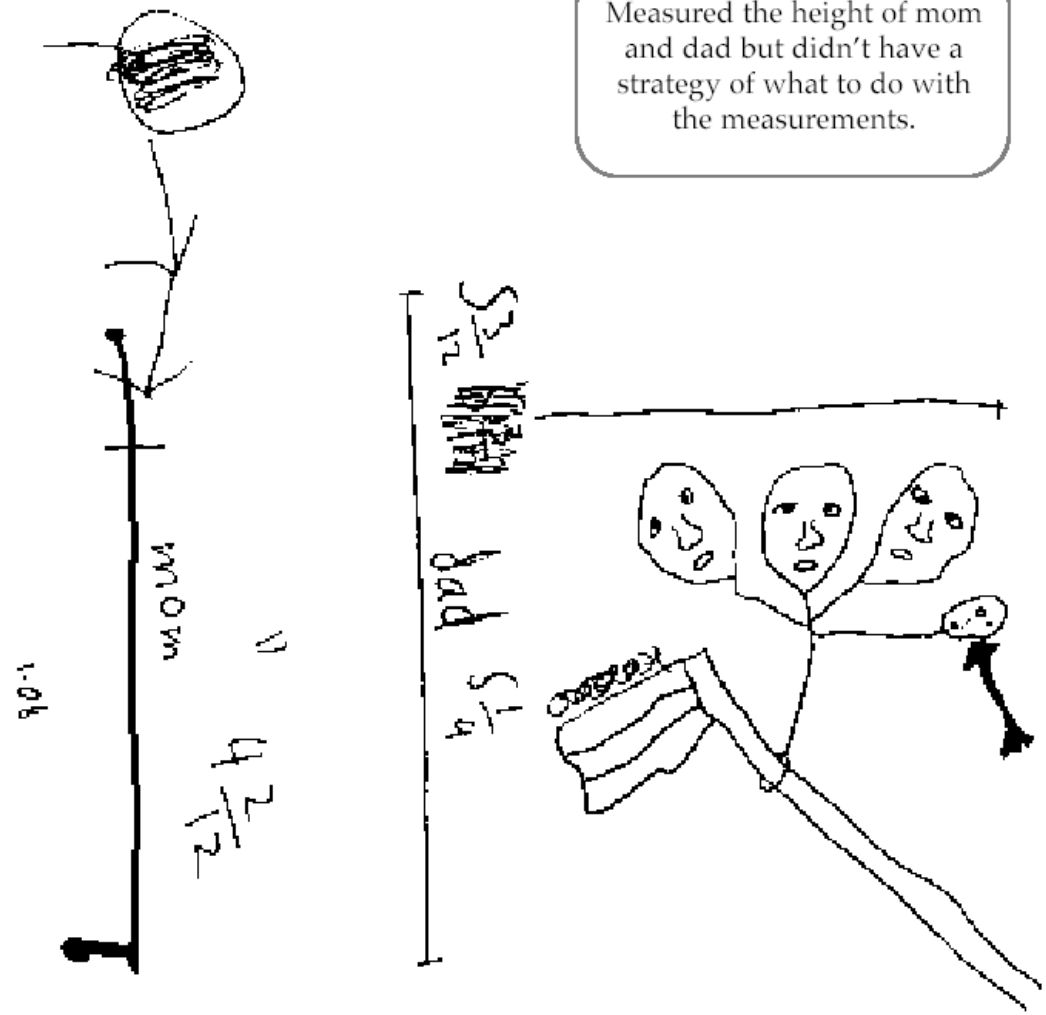
- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.

- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies, and translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

# Benchmark Papers

Novice

Measured the height of mom and dad but didn't have a strategy of what to do with the measurements.

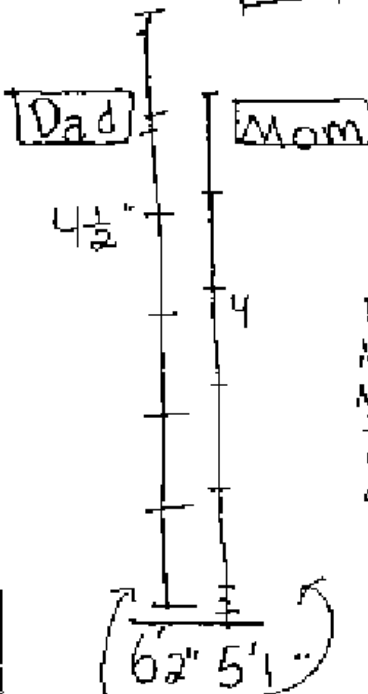


# Apprentice



$4\frac{1}{2}''$   $6'2''$   $4''$

Scale of  $\frac{3}{4}''$  of  $12\frac{3}{4}''$



Scale is wrong- Should be 12 inches. How did they get the scale?

First I measured how many inches the father was which is  $4\frac{1}{2}$  but his real height is  $6'2''$  then \*I did the same thing with the mother then I figured out there were 12 inches in a foot so I did  $12 \times 6$  because 6 is easier multiply to the  $6'2''$

$6'2''$   
 $\uparrow$  12 inches in scale  
 $\times 6$   
 $\hline 72''$   
 $+ 2''$   
 $\hline \text{Dad} = 74''$

counted out at all the the end 12 inches I added and added up two to all the even again and I got 74. Since there were 12 inches in 1 foot I

How does the student know that the left over represents 2" or 1" on the mom?  
 The work does not give me enough confidence in the student's strategy that leads up to the scale of  $\frac{3}{4}'' = 12''$

This is a good strategy to convert  $6'2''$  to  $74''$  but that is the only explanation given for the whole problem.

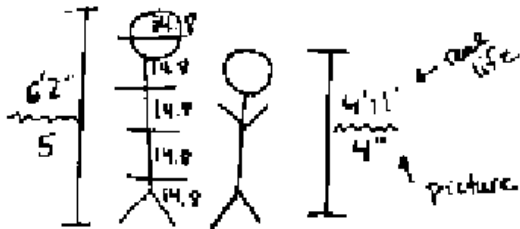
# Practitioner

Clear explanation of strategy.  
Using scale is a good strategy.



## Written Explanation

The first thing I did was measure the dad. (He was 5' tall) Then I measured the mom (she was 4' tall). After that I found out how many inches her father was (in real life) He was 74". Then I divided 74 by 5 to find out how much and inch was worth on the picture. I got 14.8". The next thing I did was multiply 14.8 by 4. (To get the moms height in inches) I got 59.2. After that I divided 59.2 by 12. (to get the moms height in feet). I got 4.9333. I rounded that to 4' 11" answer



Dad 6'2" - 5" Mom 4'11" - 4"

Good math language with the measurements labeled.

$$\begin{array}{r} 6 \times 12 = 72 \\ + 2 \\ \hline 74 \end{array} \leftarrow \text{Fathers height in inches}$$

$$74 \div 5 = 14.8 \leftarrow \text{each inch on picture is worth}$$

$$14.8 \times 4 = 59.2 \leftarrow \text{(mother's) inches in real life}$$

$$59.2 \div 12 = 4.9333 \approx 4' 11" \leftarrow \text{(mother's) feet in real life}$$

$$(9333 \div 12 \approx 11")$$

I am not sure the drawing is to an accurate scale.

## Expert

Interesting use of a fraction to multiply by 5 to get four.

This student was able to connect this problem to proportions.

She is 4" tall in the picture  
He is 5" tall in the picture and 6'2" tall in real life

Connection: I now have two ratios, which is a proportion. I now can plug her height in inches into that proportion.

$$5/6'6 = 4/$$

I multiplied 5 by  $\frac{4}{5}$  to get 4 so I have to multiply  $6\frac{6}{6}$  by  $\frac{4}{5}$  to get her height.

$$6\frac{6}{6} \times \frac{4}{5} = \frac{74}{5} = 4\frac{4}{5}$$

Now I know that she is  $4\frac{4}{5}$  feet tall, but I must convert that into inches.

$$\frac{14 \times 4 = 56 + 5 = 11\frac{1}{5}}{15 \times 4 = 60 \div 5 = 12}$$

Now I know that your mother is  $4'11\frac{1}{5}" = 4'11.2"$

General rule:  $\frac{5''}{6'6} = \frac{4''}{\text{answer}}$   $n = \text{height in picture}$

$$\text{Ex: } \frac{5''}{6'6} = \frac{4''}{\text{answer}}$$

$$6\frac{6}{6} \times \frac{4}{5} = \frac{37}{5} = 7\frac{2}{5} = 7\frac{2 \times 12}{5 \times 12} = 7\frac{24}{60} \div 5 = 7\frac{4}{5} \text{ ft tall}$$

That means that a person who measured 6 in. on the picture, would really be  $7'4\frac{4}{5}"$  tall.

Knew to convert a fraction of a foot to inches.

## Expert (cont.)

Dad is 6'2" so he is 74". He is 11.9 cm tall in the picture. Like on maps, we can find a scale, so blah cm in the picture = blah inches in real life! To find this scale, we divide 74 by 11.9. The scale in this picture is 6.22.

1 cm in this picture equals 6.22 inches in real life! So the Mom is 9.7 cm in the picture, and  $9.7 \text{ cm} \times 6.22$  in real life, which is 5 feet. (I did the problem ~~over~~ also using inches.

and I got  $4\frac{11}{5}$ ". so I am close.

Second strategy using centimeters and a more scale idea.

The student looked at both solutions and found they were close.