Teacher Instructions: The Tale of the Scale

Grade Level: 6 - 8

Task: The Tale of the Scale

Standard: Patterns, Functions, and Algebraic Concepts

Some family friends have asked you to plan a rafting expedition. A rafting company has agreed to take your group down the Babbling River (it's actually a little smaller than a river, more like a brook).

The rafting company has given you specific details on how much weight a raft can hold. A raft can safely carry the weight of 24 babies. As everyone knows, the weight of 12 babies is exactly equal to the weight of 4 teenagers; the weight of 6 teenagers is exactly equal to the weight of 3 adults.

What is the least number of rafts needed for a trip with 11 adults, 5 teenagers, and 21 babies? (Supervision of the babies is not necessary).

Context – From the Task Author: This task was given at the end of the school year when rafting for recreation is on the minds of many middle level students.

What the task accomplishes…

- Although this task may be solved in several different ways, students may apply an algebraic approach, which allows me to assess their acquisition of the application of algebraic concepts.
- Many students will also attempt this task using fractions, again allowing me to assess their competence in this area.
- This task also requires students to interpret a remainder which is a good problem solving skill to practice.

What students will do…

- Most students will make either a flow chart or diagram in which to organize the information and analyze relationships between the passengers.
- Some will then use an algebraic approach to convert all to similar units, while others will add fractions or 'fudge" the numbers to make them work.
- Most students can determine the number of boats needed for the babies and teens, but struggle with the adults.

Time Required: 1 hour

Interdisciplinary Links: The task can be linked to units on boats, shipping cargo, or to the study of rafting. Students might enjoy attempting this task after reading Trouble River by Betsy Biars, and figuring out how many rocking chairs might be transported on a raft!

Teaching Tips…

- In order to be successful with this task, students may need experiences with simple algebraic equations (ex: if 12B = 4T, what does 'T" equal?) or with adding fractions with unlike denominators.

Suggested Materials: Calculators, and manipulatives that can be used to represent babies, teens, adults and boats.

Possible Solutions…

Let: A = Adults B = Babies T = Teens

12B = 4T so 3B = T
3A = 6T so A = 2T

A = 2(3B)
A = 6B

Convert everything to babies:
11A = 66 babies
5T = 15 babies
21 babies given

THEREFORE: 66 + 15 + 21 = the equivalent of 102 babies ÷ 24 = 4 boats with the equivalent of 6 remaining babies so 5 rafts are needed.

OR

1 raft = 24 babies; 1 raft = 8 teens; and 1 raft = 4 adults THEREFORE:
(21/24 + 5/8 + 11/4) = (21/24 + 15/24 + 66/24) = (102/24) = (4 6/24) or (4 1/4) so 5 rafts are needed.

- ✓ 21/24 where one raft equals 24 babies, 5/8 where one raft equals 8 teens, and 11/4 where one raft equals 4 adults

Benchmark Descriptors:

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- The descriptors are generalizations of what student work could look like.
- It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Novice	• The novice will show little or no understanding of the task. • Little or no math language will be used, and no mathematical representation will be present. • No strategy for attempting to solve any part of the problem will be evident.
Apprentice	• The apprentice may have a strategy for solving part, but not all of the problem. • The apprentice will use little math language, and will not necessarily use mathematical representations to communicate the solution. • The apprentice will show some correct reasoning, and some work will be present.
Practitioner	• The practitioner will have a strategy for addressing all parts of the problem, and will achieve a correct answer. • The practitioner will use accurate and appropriate mathematical language and notation, and may use representations to communicate the solution.
Expert	• The expert will successfully address all parts of the problem and may have a sophisticated approach. • The expert will use precise mathematical notation and representations, as well as clearly explain the reasoning behind the decisions made. • The expert will use good problem solving strategies such as verifying her/his solution, and will make other mathematically relevant observations.

Task Specific Rubrics…

- ❖ Teachers have been given a task specific rubric with established Practitioner (proficiency) levels for each grade level of the intended task.
- ❖ The remainder of the rubric is blank so teachers have an opportunity to establish the criteria that will identify students in their class who are working at the Novice, Apprentice and Expert levels.
- ❖ Teachers may also change the intended *APS Mathematics Standards* and the established proficiency levels (which are given) to meet the needs of their individual classes.
- ❖ Teachers needing guidance for completing the rubric should refer to the **March 2001 Teacher Tip Sheet** for a more comprehensive explanation on creating task specific rubrics and developing proficiency levels. A more detailed explanation for completing the rubric is given in the **April 2001 Teacher Tip Sheet**.
- ❖ Teachers are encouraged to collaborate on the rubrics with their colleagues.
- ❖ In an effort to better serve the needs of teachers in the district, teachers may **VOLUNTARILY** return their completed rubrics for this task only, along with their students' work to RDA via interoffice mail. Student work does not have to be scored to be turned in.
  - ➢ **Please send them to: Christina Fritz at RDA – April Task and Rubric**

APS MATHEMATICAL STANDARDS…

- ❖ **The math standards stated for this task are aligned to the APS Draft Standards 2000.**

Strand – Patterns, Functions, and Algebraic Concepts:

The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focuses on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (6 – 8): The student uses tables, graphs, and symbolic representations of patterns. The student understands and uses variables and linear equations in algebraic problem solving.

Performance Standards:

Fifth Grade:

- **Uses** variables and open sentences to express simple, single-step algebraic equations (e.g., 2 + n = 5).
- **Investigates** the concept of balance in equations (e.g., 7 + 3 = 3 + x).

Sixth Grade:

- **Predicts** sequences and patterns involving varying rates of change (e.g., growth over time).
- **Analyzes** the use of variables to represent quantities (e.g., area of a rectangle: A = *lw*).
- **Explains** how expressions are used to model functions and patterns [e.g., 2, 4, 6, 8 represents $f(x) = 2n$].
- **Explains** that equations are symbolic representations of relationships, patterns, and functions.

- **Solves** one-step equations using the concept of balance when quantities are added, subtracted, or divided to both sides of an equation.

Seventh Grade:

- **Identifies** and **uses** variable expressions and formulas to solve a variety of real-life situations (e.g., Simple Interest: I =prt).
- **Represents, describes,** and **analyzes** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Develops** and **tests** strategies for solving two-step equations.
- **Translates** hypotheses into formal methods of solving algebraic equations.
- **Recognizes** and **applies** the properties of equality.

Eighth Grade:

- **Represents, describes,** and **analyzes** numerical patterns and relationships using tables, graphs, words, and standard algebraic notation.
- **Identifies** and **models** real-life situations using multiple representations.
- **Simplifies** algebraic expressions including rational expressions.
- **Investigates** and **applies** the basic mathematical properties (e.g., commutative, associative, distributive, identity, and zero) in a variety of situations.
- **Develops** and **tests** strategies for solving multi-step equations.
- **Solves** equations for specified variables (e.g., solve for *h* if A = *bh*/2).

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:

Fifth Grade:

- **Uses** fractions and decimals to help solve everyday problem.
- **Estimates** and solves problems involving addition and subtraction of fractions, and **justifies** the reasonableness of the solution.

Sixth Grade:

- **Uses** commutative, associative, identity, zero, and distributive properties when solving problems.
- **Explains** and **demonstrates** that order of operations and properties apply consistently across all math topics.
- **Selects** an appropriate operation to solve situational story problems.
- **Selects** and **uses** the appropriate number form (e.g., fraction, decimal, or percent) in a variety of situations, including measurement in U.S and metric systems.
- **Finds** Greatest Common Factor (GCF) and Least Common Multiple (LCM) using a variety of strategies, including prime factorization.

- **Translates** hypotheses into formal and fluent fractional and decimal computations using appropriate mathematical terminology.
- **Estimates** and **solves** problems involving fractions, and justifies the reasonableness of the solution.
- **Estimates** and **solves** problems involving decimals, and justifies the reasonableness of the solution.

Seventh Grade:

- **Explains** the relationship that can be expressed as ratios of part-to-whole (e.g., 5 red apples out of a total of 8 apples, expressed as 5/8).
- **Explains** the relationship that can be expressed as part-to-part (e.g., 5 red apples, 3 green apples, expressed as 5/3).

Eighth Grade:

- **Selects** the appropriate representations to describe thought provoking real-life situations.
- **Manipulates** all real numbers, their properties, and operations.
- **Develops** and **evaluates** arguments involving real numbers, their patterns and operations.
- **Develops** and **uses** strategies to estimate the results of rational-number computations and judge the reasonableness of the results.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

Performance Standards:

Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.

- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies,** and **translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

Novice

The student is able to restate the problem.

The tale of the Scale

The raft could cary the weight of 24 Bakes the weight of 12 babies is = to the weight of 4 teens. The weight of ~~two~~ 6 teens is = to the weight of 3 Adults. How many rafts are needed for a trip with 11 Adluts, 5 teens, 21 babies?

Process/Solution

I began by dividing But as I went on it got more confusing I did not get the answer.

Conclusion/Reflection

the problem was very hard.

The student ia unable to find an approach that would work.

Apprentice

It is unclear how adult rafts were found.

Problem of the week
POW.

It appears that the student used logical reasoning to find the rafts for teenagers.

Restatement -

A raft can safely carry the weight of 24 babies. The weight of 12 babies is exactly equal of 4 teenagers. The weight of 6 teenagers is equal to the weight of the adults. The questions they want to know are how many rafts are needed for a trip with 11 adults, 5 teenagers, and 21 babies? Make a diagram to show the occupants of each raft show your mathematical processincluding babies.

process and solution-

6 teenagers = 2 adults
12 babies = 4 teenagers
4 teenagers = 2 adults

5 teenagers = half raft
11 adults = 55 rafts
21 babies 1 raft

total = 7 rafts

The student is unable to find a solution using the least number of rafts.

Again, logic is used for finding rafts for babies.

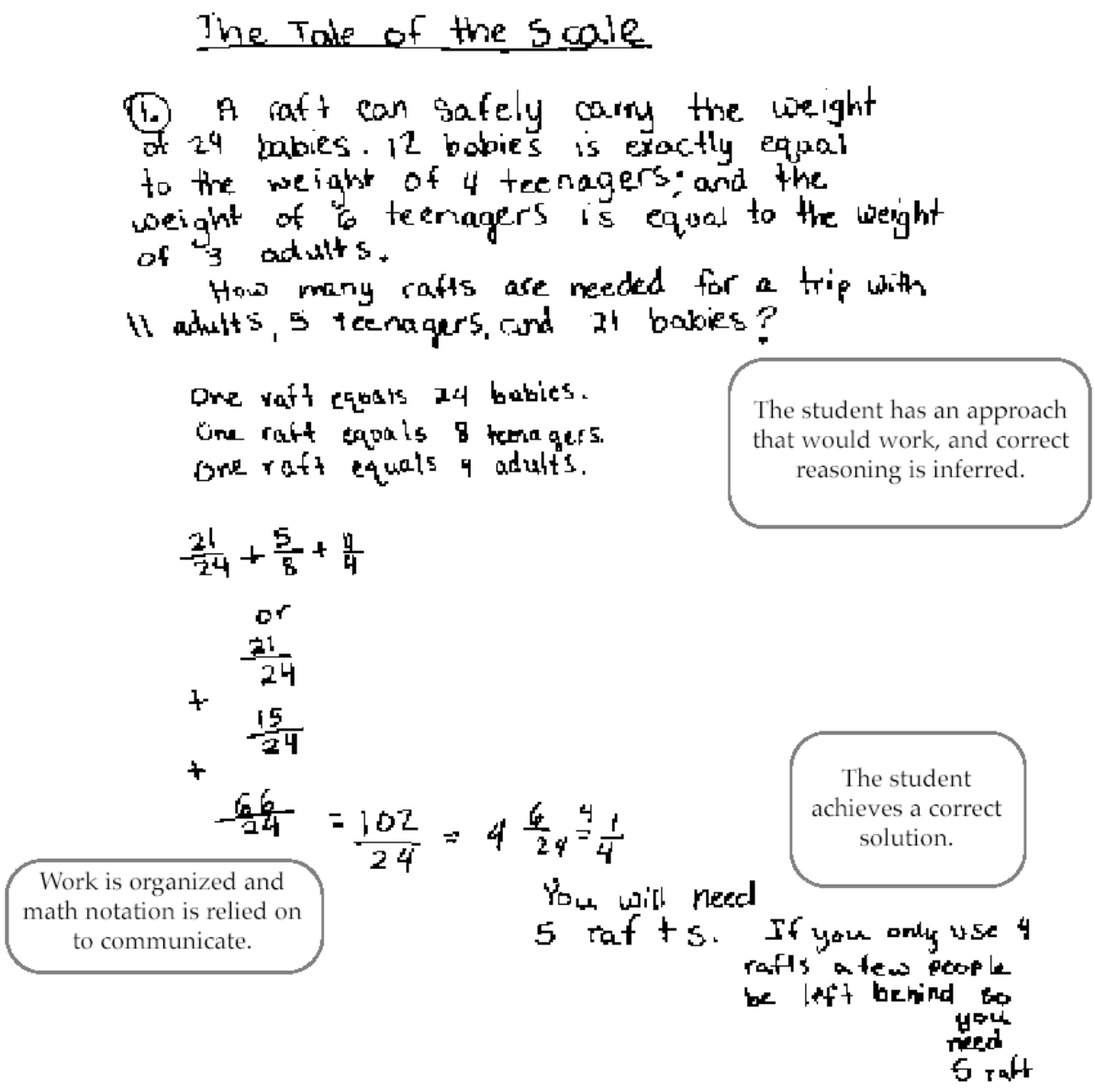
Conclusion-

for this POW I think I used adition, multiplication, dividing, but not subtracting.

Reflection -

I thought this POW was very hard because this was very complicated. for really acce
+ use your head for this POW

Practitioner



The Tale of the Scale

1. A raft can safely carry the weight of 24 babies. 12 babies is exactly equal to the weight of 4 teenagers; and the weight of 6 teenagers is equal to the weight of 3 adults.

How many rafts are needed for a trip with 11 adults, 5 teenagers, and 21 babies?

One raft equals 24 babies.
One raft equals 8 teenagers.
One raft equals 4 adults.

$\frac{21}{24} + \frac{5}{8} + \frac{11}{4}$

or

$\frac{21}{24}$

+

$\frac{15}{24}$

+

$\frac{66}{24} = \frac{102}{24} = 4\frac{6}{24} = 4\frac{1}{4}$

You will need 5 rafts. If you only use 4 rafts a few people be left behind so you need 5 raft

The student has an approach that would work, and correct reasoning is inferred.

The student achieves a correct solution.

Work is organized and math notation is relied on to communicate.