

# Teacher Instructions: Stained-Glass Window Designs

**Grade Level:** 6 - 8

**Task:** Stained – Glass Window Designs

**Standard:** Patterns, Functions, and Algebraic Concepts

You are planning a stained-glass window section. You start with one window and you divide it in two sections. But this isn't intricate enough for you so you divide each of these sections in two sections again. You do this many times.

How many sections do you have after 10 divisions?

After 20 divisions?

Remember, I will want to know all of the steps you take along the way. Can you make any connections or discuss any patterns you see?

**Context – From the Task Author:** My sixth graders just finished a unit on multiplying that included a section on exponents. I wanted to give them a problem that involved exponents to see how many would be able to recognize the pattern and could make a generalization using exponents.

## What the task accomplishes...

- I wanted the task to force students to keep track of the data in a chart form. From the chart I was hoping they would at least see the doubling pattern and the most astute would recognize the pattern as the powers of two.
- I was also thinking they might connect this task with an exercise we did of folding papers and seeing the layers were increasing by the powers of two.

## What students will do...

- Most students started with drawing a square and dividing the square.
- Many did not keep track of the number of sections after each division and had to start over.
- Some jumped to the wrong conclusion and thought the pattern was a geometric sequence instead of an exponential sequence.

**Time Required:** It took 45 minutes to solve the task. After about 25 minutes, I gave students a 5" X5" piece of paper to do a stained-glass design after solving the problem. Some used this piece of paper to start over and keep track of the sections after each division.

**Interdisciplinary Links:** I asked the French teacher to come in and show pictures to the class of famous stained-glass windows in cathedrals, and how the windows told a story in the glass. The students also designed their own stained glass windows that I put together as a quilt that made a nice decoration.

## Teaching Tips...

- Some of my students did not pick up on the word EACH in the problem. When they divided their squares they were only dividing one section each time.
- Some had a hard time dividing each section after the third, or fourth division.

- Many realized the importance of keeping track of the numbers as they divided the sections so they could look for a pattern after awhile and not have to continue dividing sections and counting.
- The stained glass window they make on their own could be any design, it did not have to follow the rules of the problem.

**Suggested Materials:** I always ask at the beginning of a problem, if anyone wants any tools to help them solve the problem. I don't like just handing out materials that might direct or redirect their own approaches. Students asked for rulers, unlined paper, protractors, compasses, and only a few asked for calculators.

**Possible Solution...**

- ✓ First you have one section ( $2^0$ ) then you have 2 sections ( $2^1$ ), then 4 sections ( $2^2$ ), then 8 sections ( $2^3$ ).
- ✓ The 10<sup>th</sup> division is  $2^{10} = 1024$  sections and the 20<sup>th</sup> division gives you  $2^{20} = 1,048,576$  sections.

**Benchmark Descriptors:**

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- The descriptors are generalizations of what student work could look like.
- It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

**Novice**

- ✓ A novice may misunderstand the problem and not divide all sections each time.
- ✓ The novice may also jump to the conclusion that it is a geometric progression (2, 4, 6, 8...) instead of an exponential progression.
- ✓ The novice will use little of no math language and will probably not organize his/her data into chart or table.

**Apprentice**

- ✓ An apprentice may understand the problem, but may not have an approach to completely answer the questions.
- ✓ The apprentice may not be able to find the 10<sup>th</sup> and 20<sup>th</sup> term, or may make some computational error.
- ✓ Some math language may be used and a math representation will be attempted.

**Practitioner**

- ✓ A practitioner will successfully divide each section into two sections, as well as complete the other mathematics required by the task.
- ✓ Practitioners may comment on the fact that the number of sections doubles each time.
- ✓ They may have a mathematical representation in a diagram of the stained-glass window in the beginning of the divisions and/or have a chart that shows the pattern.
- ✓ Mathematics language will be used to communicate the solution.

### Expert

- ✓ An expert solution will have all of the qualities described in a practitioner's solution, but also will relate this number sequence to exponents.
- ✓ Experts may also be able to generalize the solution algebraically.

### APS Mathematical Standards...

❖ The math standards stated for this task are aligned to the APS Draft Standards 2000.

#### Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

**Benchmark (6 – 8):** The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

#### Performance Standards:

Seventh Grade:

- **Explain and model** the value of exponents and square roots.
- **Simplify and evaluate** (solve) numerical expressions involving exponents (e.g.  $2^3 = 2 \times 2 \times 2 = 8$ ).

Eighth Grade:

- **Simplify and evaluate** (solve if possible) algebraic expressions for all types of real numbers including exponents and common square roots.

#### Strand - Geometry, Spatial Sense, and Measurement:

Students will demonstrate an understanding of concepts, properties, and relationships of geometry and measurement through experiences with meaningful mathematical problems, while focusing on identifying, describing, classifying, visualizing, comparing, estimating, and measuring various aspects of shapes and sizes.

**Benchmark (6 – 8):** The student will understand the relationships between 2- and 3-dimensional shapes and identify, build and transform shapes. The student will use inductive and deductive arguments to solve problems. The student will use metric and customary measurement systems and select the appropriate measurement unit for a given situation.

#### Performance Standards:

Sixth Grade:

- **Draw and explain** congruent 2-dimensional figures using mathematical terminology.

## **Strand – Patterns, Functions, and Algebraic Concepts:**

Students will demonstrate an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

**Benchmark (6 – 8):** The student will use tables, graphs, and symbolic representations of patterns. The student will understand and use variable and linear equations in algebraic problem solving.

### **Performance Standards:**

Sixth Grade:

- **Predict** sequences and patterns involving varying rates of change.
- **Explain** how expressions are used to model functions and patterns.

Seventh Grade:

- **Identify and use** variable expressions and formulas to solve a variety of real-life situations.
- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.

Eighth Grade:

- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Identify and model** real-life situations using multiple representations.

## **Strand - Global Mathematical Processes:**

Students will understand and use mathematical process.

**Benchmark (K - 12):** The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

### **Performance Standards:**

Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.

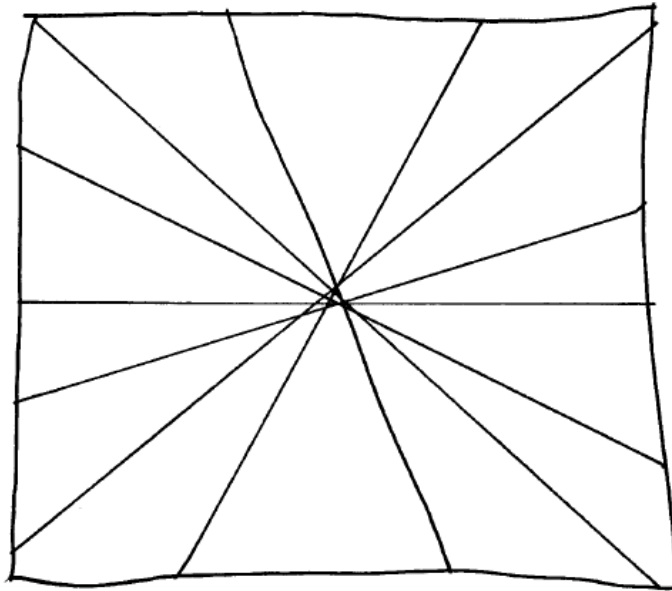
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies, and translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

# Benchmark Papers

## Novice

The student misunderstood the task and is only dividing two sections each time. Each time they divide the window they use only one line.

D section  
 $1 \times 2 = 2$   
 $2 \times 2 = 4$   
 $3 \times 2 = 6$   
 $4 \times 2 = 8$   
 $5 \times 2 = 10$   
 $6 \times 2 = 12$   
 $7 \times 2 = 14$   
 $8 \times 2 = 16$   
 $9 \times 2 = 18$   
 $10 \times 2 = 20$   
 you times 2  
 to the number  
 of dimensions.



D 5  
 $49 \times 2 = 198$   
 $54 \times 2 = 108$   
 $80 \times 2 = 160$   
 $20 \times 2 = 40$   
 $30 \times 2 = 60$   
 $40 \times 2 = 80$   
 $50 \times 2 = 100$   
 $60 \times 2 = 120$   
 $70 \times 2 = 140$   
 $80 \times 2 = 160$   
 $90 \times 2 = 180$   
 $100 \times 2 = 200$

Equation  
 $\downarrow$   
 $(D \times 2 = 5)$

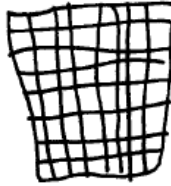
$352 \times 2 = 704$

The student has generated a geometric progression instead of an exponential progression.

An attempt at algebra. However, the equation is wrong and the student doesn't define 'D' or 'S'.

# Apprentice

Chart	# of Division	# of sections
	1st	2
10th 1,024	2nd	4
20th 1,480,576	3rd	8
	4th	16
	5th	32
	6th	64
	7th	128
	8th	256
	9th	512
	10th	1,024
	11th	2,048



## Observation #1

You take the answer to the previous question and multiply it by 2.

## Observation #2

all #'s are even ones

Good observation.

The student correctly found the tenth term, but has made a mathematical error somewhere and the twentieth term is incorrect.

$$10 = 1,024$$

$$20 = 1,480,576$$

x = answer to previous question

x = 2 (constant)

A = 1st question

$$xy = A^x$$

eg.  $4 \times 2 = 8$

$$\begin{array}{r} 32,768 \\ \underline{2} \\ 65,536 \\ \underline{2} \\ 131,072 \\ \underline{2} \\ 262,144 \end{array}$$

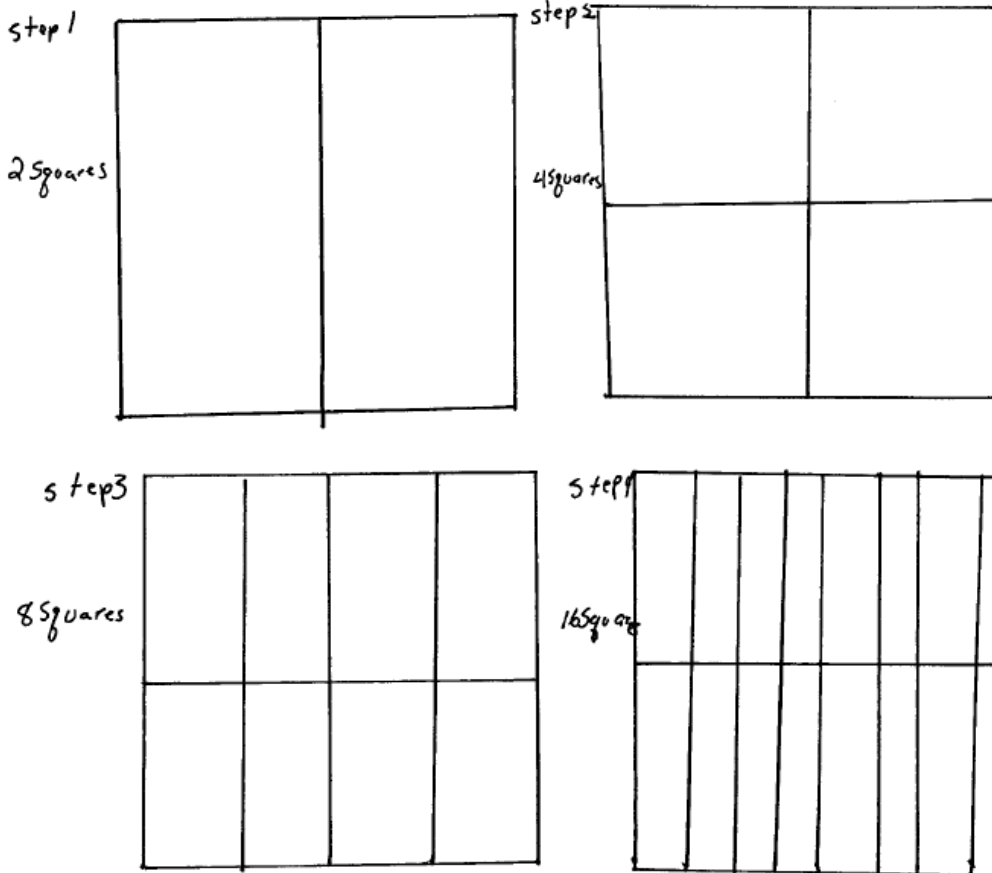
$$\begin{array}{r} 162,144 \\ \underline{2} \\ 324,288 \\ \underline{2} \\ 648,576 \end{array}$$

(continued)

# of Divi	# of sections
12	4,096
13	8,192
14	16,384
15	32,768
16	65,536
17	131,072
18	262,144
19	524,288
20	1,048,576

The student made an attempt at a generalization, however, it is not a generalization it is simply explaining a pattern using too many variables.

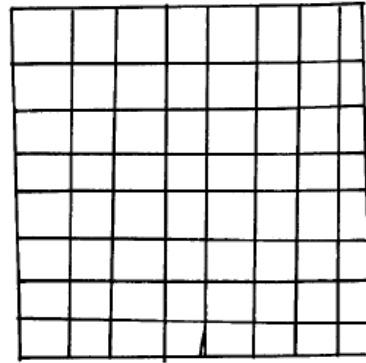
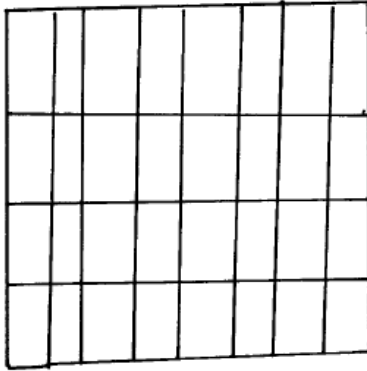
# Practitioner



Great use of a diagram to show progression.

# Practitioner (cont.)

Effective use of charts.



Beginning example		Step	Square #
Step	Square #		
1	2	7	128
2	4	8	256
3	8	9	512
4	16	10	1024
5	32	11	2048
6	64	12	4096
		13	8192
		14	16384
		15	32768
		16	65536
		17	131072
		18	262144
		19	524288
		20	1048576

*You multiply the # you have by 2 and you keep going that:*

$$N \times 2 = N$$

An attempt at a generalization, however, it is just trying to explain the pattern and the variable 'N' is not defined.

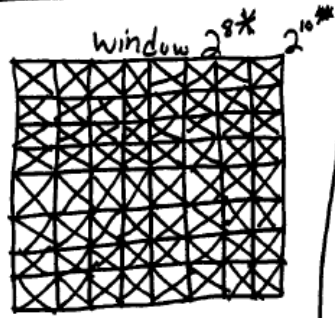
# Expert

Nice chart that connects the number of sections and exponents.

Exponents of 2

Exponent	Standard form (N)
2 <sup>1</sup>	2
2 <sup>2</sup>	4
2 <sup>3</sup>	8
2 <sup>4</sup>	16
2 <sup>5</sup>	32
2 <sup>6</sup>	64
2 <sup>7</sup>	128
2 <sup>8</sup>	256
2 <sup>9</sup>	512
2 <sup>10</sup>	1024
2 <sup>11</sup>	2048
2 <sup>12</sup>	4096
2 <sup>13</sup>	8192
2 <sup>14</sup>	16384
2 <sup>15</sup>	32768
2 <sup>16</sup>	65536
2 <sup>17</sup>	131072
2 <sup>18</sup>	262144
2 <sup>19</sup>	524288
2 <sup>20</sup>	1,048,576

Rule \*\*\*  
 $2^x = N$   
 x = number of divisions  
 N = Number of sections



\*\*\* multiply the base (2) times itself of times the exponent identifies.  
 e.g.  $2^8 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$   
 The answer will be the number of sections or (n).

Nice example of 2<sup>8</sup> in the diagram.

$2^{10} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 1024$

4 · 4 · 4 · 4 · 4

16 · 16 · 4

100    20    20

46    136    28

4

\*\*\* 1024

Good verification of arithmetic.

\*\*\* use chart  
 connection this problem deals with exponents