Teacher Instructions: Sink or Swim?

Grade Level: 6 - 8

Task: Sink or Swim?

Standard: Patterns, Functions, and Algebraic Concepts

In New Zealand there is a river known as the Kaituna. On it, there exists a 23-foot waterfall; some people like to go over the waterfall on a raft. A group of 6 people decide to give it a try; they consist of four adults and two teenagers.

The raft MUST be traveling at least 30 miles per hour in order to successfully go over the falls; otherwise it will tip over. The river flows at one-sixth of the needed speed. An adult can paddle the raft at a speed of 5 miles per hour. Three teenagers can paddle the same speed as two adults.

How fast is the raft traveling?
Will the group be able to make it over the falls without tipping over?

Context – From the Task Author: This task was given at the end of the school year when rafting for recreation is on the minds of many middle level students.

What the task accomplishes…

- This task allows the teacher to see the students' command of fractions and decimals, and how students deal with repeated decimals.

What students will do…

- Most students will start with what is easy, and work on the more challenging parts progressively. For instance, most students will easily see that the adults can paddle a total of 20 mph, but will then be slightly more challenged with finding the 1/6 of the river speed.
- The greatest challenge is in determining the rate of the teenagers, and students will come up with several different ways of doing so.
- Finally students will interpret the remainder that results from their calculations.

Time Required: 1 hour

Interdisciplinary Links: The task can be linked to units on boats, rivers, or to the recreational study of rafting. Students might enjoy attempting this task after reading Trouble River by Betsy Byars, and figuring out how long it might take the main character and his grandmother to travel down river.

Teaching Tips…

- In order to be successful with this task, students need experience with dividing whole numbers by fractional amounts, and with multiplying and dividing decimals.
- Students will also need experience with interpreting remainders.

Suggested Materials: Calculators

Possible Solution…

- ✓ River travels 1/6 of 30 or 5 mph.
- ✓ Adults paddle 5 mph x 4 adults = 20 mph.
- ✓ 3 teenagers paddle the equivalent of 2 adults or 2(5) = 10 mph.
- ✓ 1 teenager paddles (10 mph. ÷ 3) = 3.333, so 2 teenagers paddle 2 x 3.333 or 6.666 mph.

River: 5
Adults: 20
Teens: + 6.666
31.666 mph

- ✓ The raft travels over the falls without trouble, since their speed is greater than 30 mph.

Benchmark Descriptors:

- ❑ The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- ❑ The descriptors are generalizations of what student work could look like.
- ❑ It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- ❑ It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Novice

- ✓ The novice will show little or no understanding of the task.
- ✓ Little of no math language will be used and no mathematical representation will be present.
- ✓ No strategy for attempting to solve any part of the problem will be evident.

Apprentice

- ✓ The apprentice may have a strategy for solving part but not the entire problem.
- ✓ The apprentice will use little math language and will not necessarily use mathematical representations to communicate the solution.
- ✓ The apprentice will show some correct reasoning, and some work will be present.

Practitioner

- ✓ The practitioner will have a strategy for addressing all parts of the problem and will achieve a correct answer.
- ✓ The practitioner will use accurate and appropriate mathematical language and notation and may use representations to communicate the solution.

Expert

- ✓ The expert will successfully address all parts of the problem and may have a sophisticated approach.
- ✓ The expert will use precise mathematical notation and representations, as well as clearly explain the reasoning behind the decisions made.
- ✓ The expert will use good problem solving strategies such as verifying his/her solution and will make other mathematically relevant observations.

APS Mathematical Standards…

❖ **The math standards stated for this task are aligned to the APS Draft Standards 2000.**

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:

Sixth Grade:

- **Select** an appropriate operation to solve situational story problems.
- **Select and use** the appropriate number form (fraction, decimal, or percent) in a variety of situations, including measurement in U.S. and metric systems.
- **Develop and test** strategies for adding and subtracting fractions with like and unlike denominators.
- **Develop and test** strategies for multiplying and dividing fractions.
- **Develop and test** strategies for adding and subtracting decimals.
- **Develop and test** strategies for multiplying and dividing decimals.
- **Estimate and solve** problems involving fractions & decimals, and **justify** the reasonableness of the solution.

Seventh Grade:

- **Translate** problem-solving strategies into efficient computation using appropriate mathematical terminology.

Eighth Grade:

- **Select** the appropriate representations to describe thought provoking real-life situations.
- **Manipulate** all real numbers, their properties, and operations.
- **Develop and evaluate** arguments involving real numbers, their patterns and operations.
- **Develop and use** strategies to estimate the results of rational-number computations and judge the reasonableness of the results.

Strand – Patterns, Functions, and Algebraic Concepts:

Students will demonstrate an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (6 – 8): The student will use tables, graphs, and symbolic representations of patterns. The student will understand and use variable and linear equations in algebraic problem solving.

Performance Standards:
Sixth Grade:

- **Solve** one-step equations using the concept of balance when quantities are added, subtracted, or divided by both sides of an equation.

Seventh Grade:

- **Identify and use** variable expressions and formulas to solve a variety of real-life situations.
- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Translate** hypotheses into formal methods of solving algebraic equations.

Eighth Grade:

- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Identify and model** real-life situations using multiple representations.
- **Develop and test** strategies for solving multi-step equations.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

Performance Standards:
Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).

- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies,** and **translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

Novice

The students did not achieve a correct answer.

They were going 30 miles per hour.

They are not going to make it over they did not have erough speed.

There is no work to support this answer.

we did not show work because we were not shure

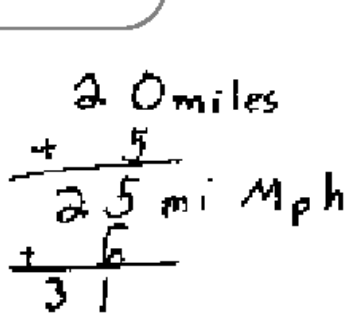
Students communicate their lack of assuredness in their solution.

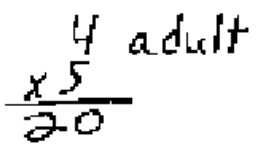
Apprentice

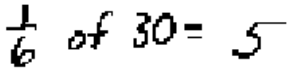
Some math language is used correctly and some incorrectly.

The student accurately finds the rate of adults.

The student accurately finds rate of river.







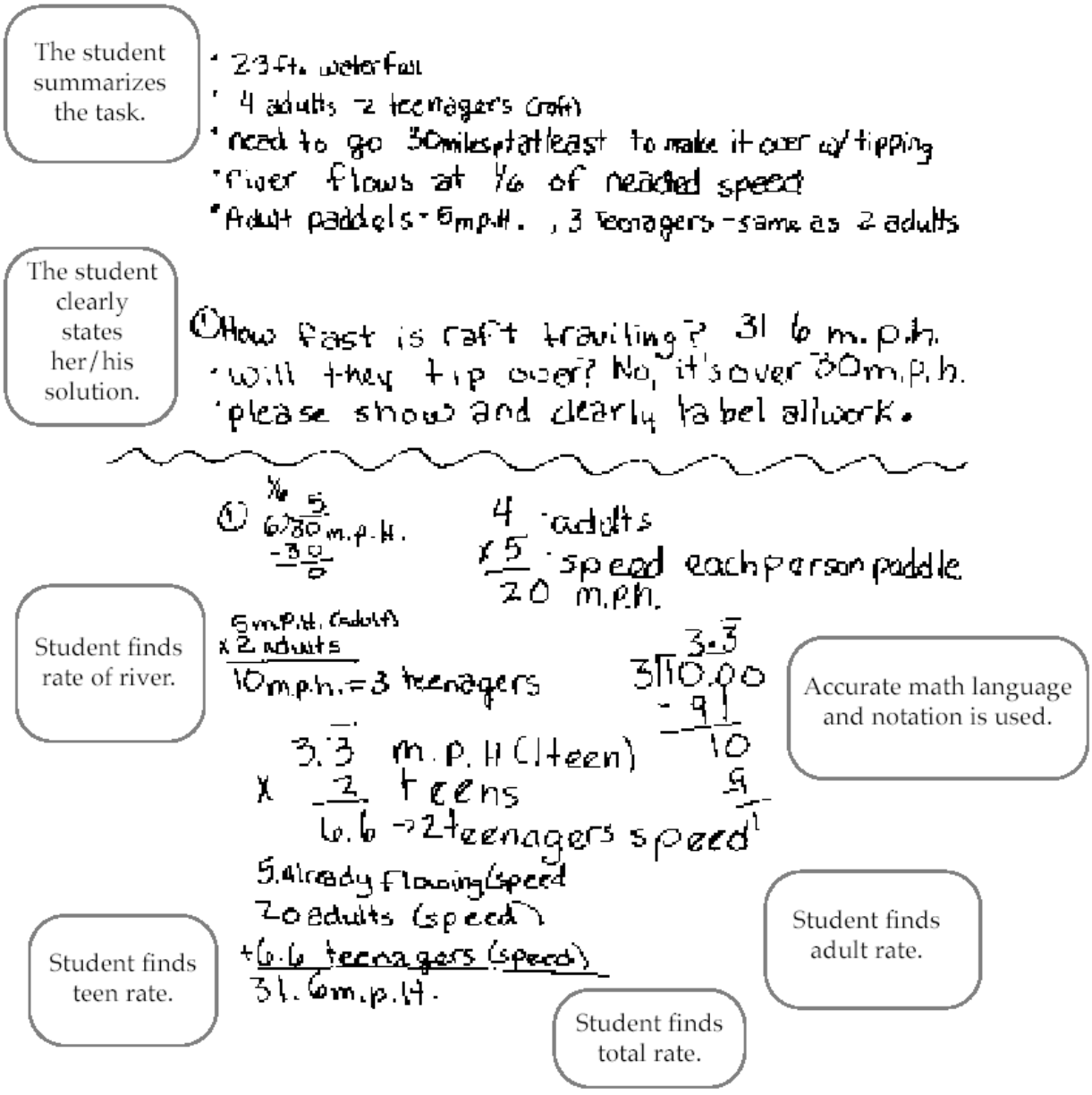
1. The raft will travel about 31 Mph.

2. The raft and the people will not tip.

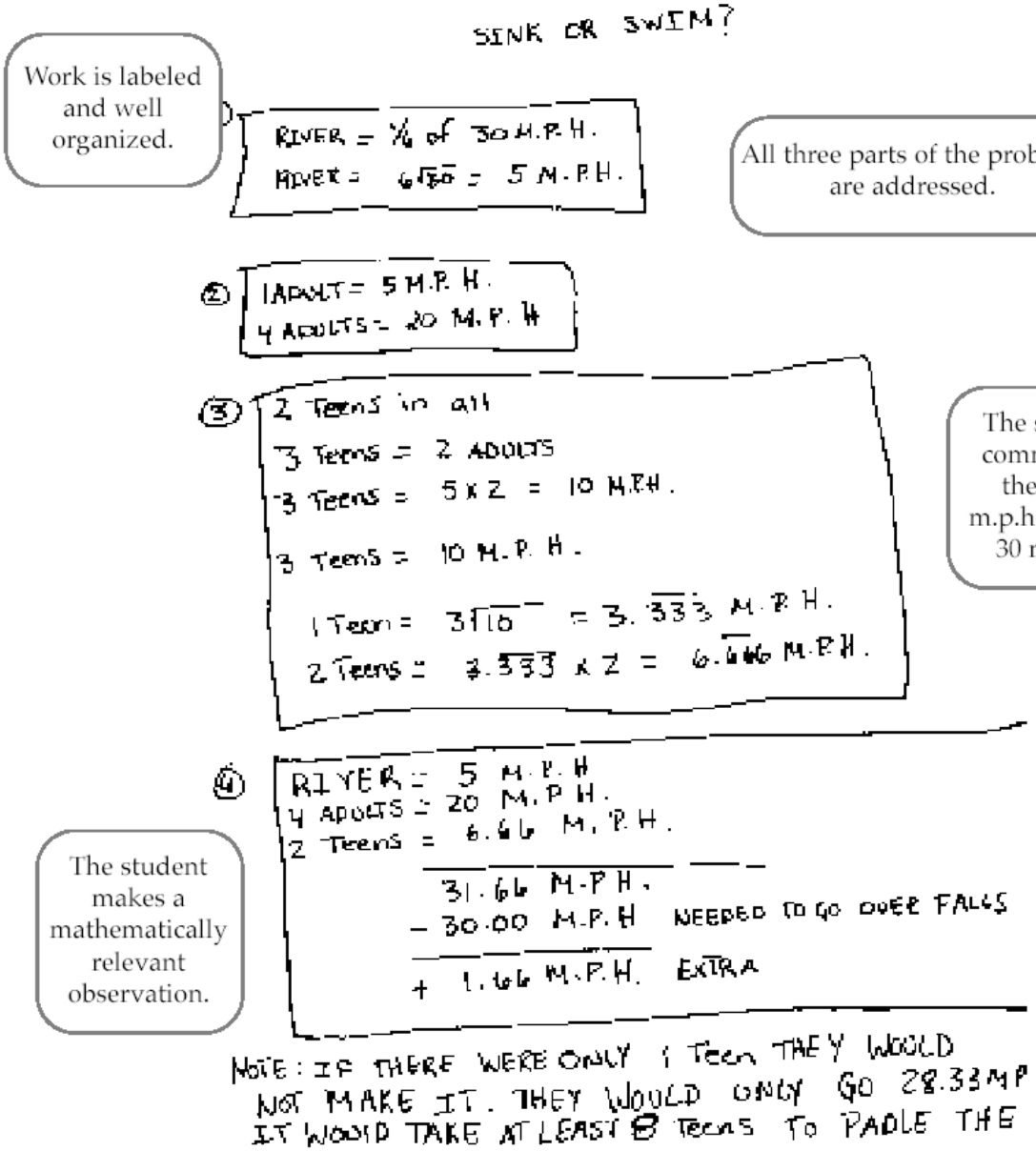
It is unclear how the student came to a rate of 6mph, presumably for the teens. This is also incorrect.

The "answer" is technically correct, but the work doesn't support the answer.

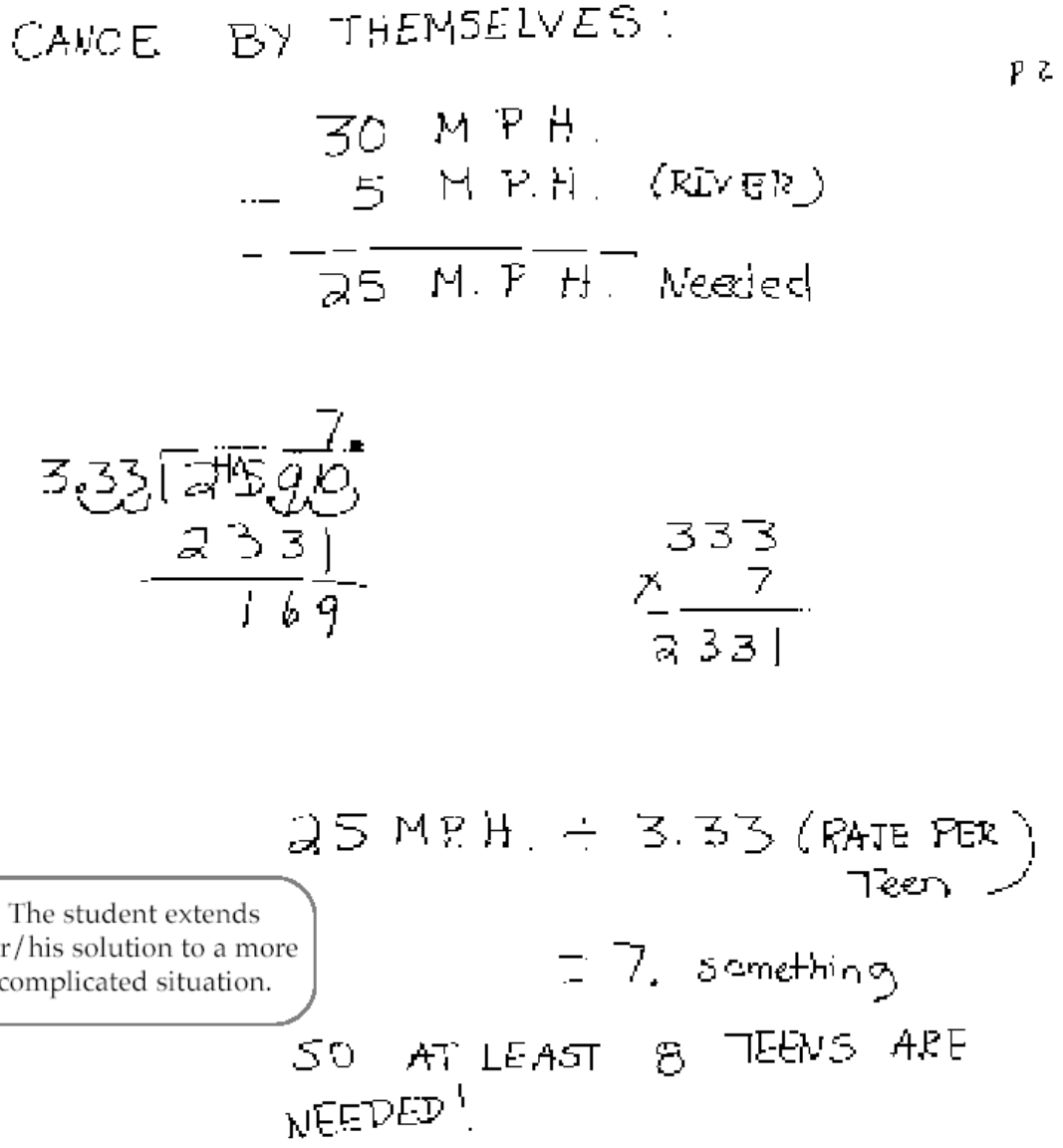
Practitioner



Expert


SINK OR SWIM?
Work is labeled and well organized.
RIVER = ½ of 30 M.P.H.
RIVER = 6)30 = 5 M.P.H.
All three parts of the problem are addressed.
② 1 ADULT = 5 M.P.H.
4 ADULTS = 20 M.P.H
③ 2 Teens in all
3 Teens = 2 ADULTS
3 Teens = 5 x 2 = 10 M.P.H.
3 Teens = 10 M.P.H.
1 Teen = 3)10 = 3.333 M.P.H.
2 Teens = 3.333 x 2 = 6.666 M.P.H.
The student comments on the extra m.p.h. beyond 30 m.p.h..
④ RIVER = 5 M.P.H
4 ADULTS = 20 M.P.H.
2 Teens = 6.66 M.P.H.
31.66 M.P.H.
- 30.00 M.P.H NEEDED TO GO OVER FALLS
+ 1.66 M.P.H. EXTRA
The student makes a mathematically relevant observation.
NOTE: IF THERE WERE ONLY 1 Teen THEY WOULD NOT MAKE IT. THEY WOULD ONLY GO 28.33 MP IT WOULD TAKE AT LEAST 2 Teens TO PADLE THE

Expert (cont.)



The student extends her/his solution to a more complicated situation.