Teacher Instructions: The Double Scoop Dilemma

Grade Level: 6 - 8

Task: The Double Scoop Dilemma

Standard: Patterns, Functions, and Algebraic Concepts

The Ice Cream Shop serves 40 different flavors of ice cream.
How many different double-scoop cones can they make?

Context – From the Task Author: This task was given to students while studying combinations and permutations. It was spring, and ice cream season was just around the comer, making this task a yummy motivation.

What the task accomplishes…

- This task was given in correlation with the ***Exemplars*** task "Stir Crazy" and was used to compare and contrast combinations and permutations.
- After solving the task students can discuss, or even write a reaction in their math journals about the patterns involved with combinations and permutations.

What students will do…

- Students will employ a variety of problem-solving strategies. One of the most effective is solving a simpler version of the problem and then applying the knowledge learned to the 40 flavors of ice cream.
- Some students will apply previous knowledge about tasks dealing with combinations, while others might try drawing diagrams or creating systematic lists.

Time Required: One, 45-minute period.

Interdisciplinary Links: This task links to the study of ice cream. Students could learn how it is made, what it is made of, and even make some themselves!

Teaching Tips…

- One might adapt the task to make it simpler for students who have special needs, by changing the number of ice cream flavors.
- One could also make the task more complicated by making the students find the possible number of triple or quadruple-scoop cones.

Suggested Materials: Calculators

Possible Solution…

- ✓ The correct mathematical formula to apply is $\{n(n\text{-}1)\}$ with n equal to the number of flavors, so the correct solution would be 40(39) = 1560 possible double-scoop cones.

Benchmark Descriptors:

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- The descriptors are generalizations of what student work could look like.
- It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.

- It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Novice

- ✓ The novice will not have an approach that will work for solving the task.
- ✓ S/he will think that 40 flavors multiplied by 40 flavors will equal 80 cones, but does not consider that for instance chocolate / vanilla and vanilla / chocolate are the same cones.
- ✓ Little or no math language will be used, but some rudimentary diagrams may be attempted.

Apprentice

- ✓ The apprentice will have a partially correct solution or a strategy that could lead to a solution.
- ✓ S/he may recognize that chocolate / vanilla and vanilla / chocolate are the same cones, but may not be able to apply that knowledge to get a correct solution.
- ✓ It may be difficult to follow the student's approach, and some reasoning will be flawed.
- ✓ Some correct math language will be used and math representations may be attempted, although computation errors may also be present.

Practitioner

- ✓ The practitioner will have a correct solution with supporting work.
- ✓ S/he will develop a rule or notice a pattern for solving the task.
- ✓ This student will use accurate and appropriate math language and representations.
- ✓ All work will be shown and it will be clear that the student did to solve the task.

Expert

- ✓ The expert will create a formula for solving the task that will work no matter how many flavors he or she is presented with.
- ✓ Accurate and appropriate algebraic and other notations will be used.
- ✓ The expert will extend the solution by making mathematically relevant comments or observations about the solution.

APS Mathematical Standards…

- ❖ **The math standards stated for this task are aligned to the APS Draft Standards 2000.**

Strand – Patterns, Functions, and Algebraic Concepts:

Students will demonstrate an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (6 – 8): The student will use tables, graphs, and symbolic representations of patterns. The student will understand and use variable and linear equations in algebraic problem solving.

Performance Standards:

Sixth Grade:

- **Compare and contrast** models of discrete functions and continuous functions in real-life applications.
- **Analyze** the use of variables to represent quantities.
- **Explain** how expressions are used to model functions and patterns.

Seventh Grade:

- **Identify and use** variable expressions and formulas to solve a variety of real-life situations.
- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Translate** hypotheses into formal methods of solving algebraic equations.

Eighth Grade:

- **Represent, describe, and analyze** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.
- **Identify and model** real-life situations using multiple representations.
- **Develop and test** strategies for solving multi-step equations.

Strand – Data Analysis, Statistics, and Probability

Students will identify patterns and special features of data and events of chance through experiences with meaningful mathematical problems while focusing on comparing, predicting, representing data, and making decisions to communicate mathematical understanding.

Benchmark (6 – 8): The student will design a data question with two variables and collect, represent and analyze the data. The student will use a variety of graphical representations to display data and understand measures of center and spread. The student will make conjectures and compute simple probability outcomes using a variety of tools.

Performance Standards:
Seventh Grade:

- **Apply** counting principles to determine sample space (e.g., tree diagrams, fundamental counting principle, combinations, and permutations).

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:
Seventh Grade:

- **Translate** problem-solving strategies into efficient computation using appropriate mathematical terminology.

Eighth Grade:

- **Simplify and evaluate** (solve if possible) algebraic expressions for all types of real numbers including exponents and common square roots.
- **Examine, describe and model** exponential patterns that reflect growth and decay.
- **Develop and evaluate** arguments involving real numbers, their patterns and operations.
- **Develop and use** strategies to estimate the results of rational-number computations and judge the reasonableness of the results.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

Performance Standards:
Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies,** and **translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

Novice

I thought this problem was easy but it isn't. I think 40 x 40 = 1600 but my frends got different. I dont know how. I am 40 off them.

The student lacks an approach that will lead to a correct answer.

Some parts are clear and some math language is used.

At least the student knows that he or she is incorrect.

Apprentice

1
2-1
3 1-2
4-1-2-3
5-1-2-3
6-1-2-3
7-1-2-3
8-1-2-3
9-1-2-3
10-1-2-3
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31-1-2
32-1-2
33-1-2
34-1-2
35-1-2
36-1-2
37-1-2
38 1-2
39-1-2
40-1-2

This will take too long.

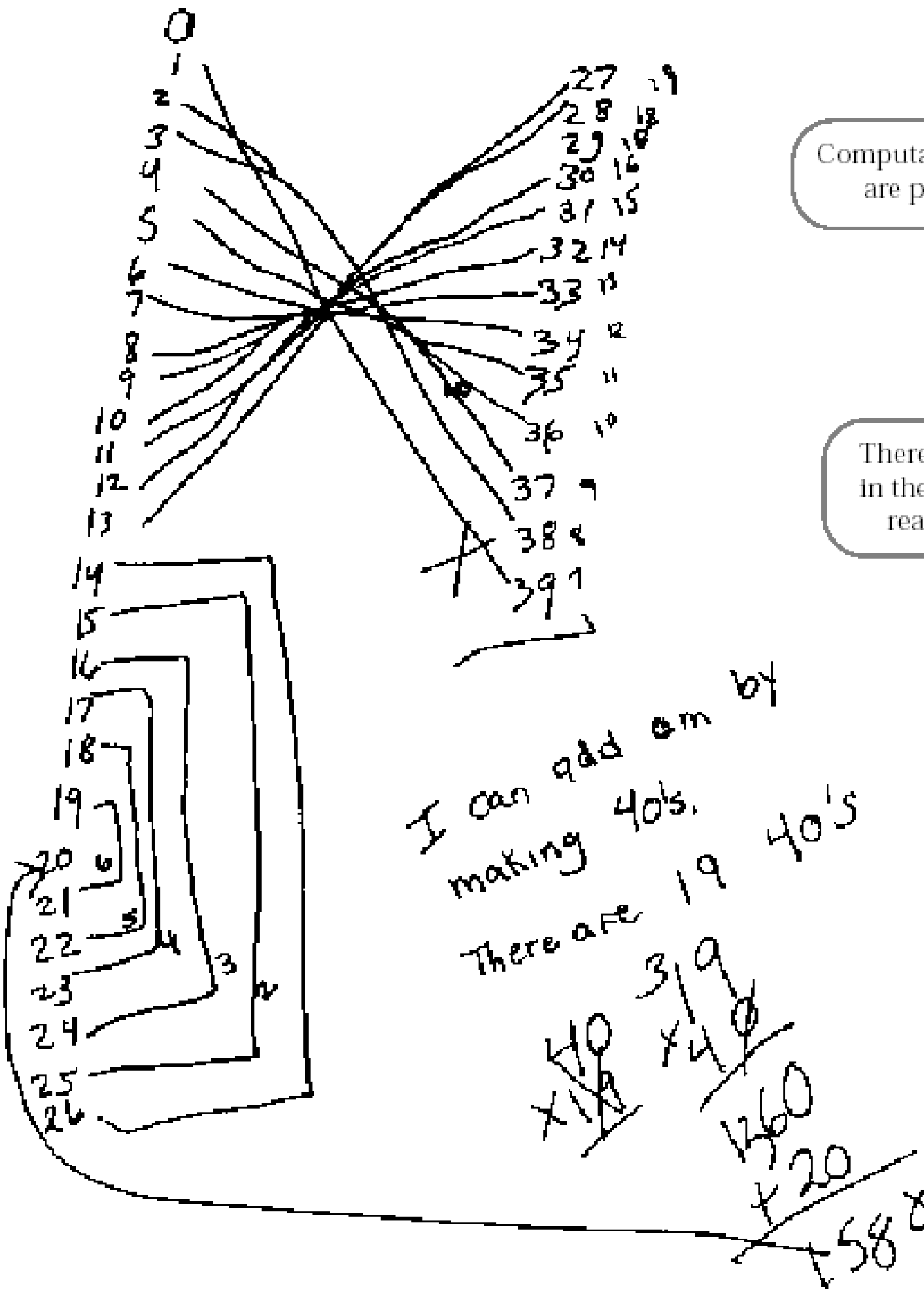
I noticed
1 has none
2 has 1
3 has 2
4 has 3
so 40 probly has 39 then you should add them up →

This attempt at a representation could be made more clear.

The student has an approach that would work if carried through.

Some correct reasoning is evident.

Apprentice (cont.)



Computation errors are present.

There is a flaw in the student's reasoning.

Practitioner

MY SOLTUION

I THINK I WILL FIRST TRY MAKING THE PROBLEM MORE SIMPLE AND SOLVING IT. IF I HAD 4 FLAVORS INSTEAD OF 40 I COULD MAKE

BOTTOM SCOOP:	CHOC	VANILLA	STAWBERRY	CHOC CHIP
TOP SCOOP:	CHOC	CHOC	CHOC	CHOC
POSSIBLITIES	VANILLA	VANILLA	VANILLA	VANILLA
	STRAW	STRAW	STRAW	STRAW
	CC	CC	CC	CC

THERE WOULD BE 16 DIFFERENT CONES. BUT CHOC-VANILLA IS THE SAME AS VANILLA-CHOC SO I WILL TAKE ONE OUT OF EACH COLUMN AND THAT LEAVES 12 COMBINATIONS.

BOTTOM SCOOP:	CHOC	VANILLA	STAWBERRY	CHOC CHIP
TOP SCOOP:	CHOC	CHOC	CHOC	CHOC
POSSIBLITIES	VANILLA	VANILLA	VANILLA	VANILLA
	STRAW	STRAW	STRAW	STRAW
	CC	CC	CC	CC

SO AT FIRST I THOUGHT THAT 4 FLAVORS X 4 FLAVORS = 16 CONES. BUT I WAS WRONG. I SHOULD HAVE DONE 4 FLAVORS X 3 FLAVORS SO THAT CONES DIDN'T REPEAT. THAT EQUALS 12 CONES. SO NOW I KNOW THAT 40 FLAVORS X 39 FLAVORS = 1560 DIFFERNT CONES!

The representations are supported by text.

A correct answer is achieved.

The student's approach and reasoning are explained.

Expert

Double Scoop Dilemma

This problem is like another problem I did called the Handshake problem. It said if there were 5 kids and each shook each the other students hands, how many handshakes would there be in all. In that problem I used the formula

$$\frac{x(x-1)}{2} \qquad x = \# \text{ of flavors}$$

This is the same kind of problem and you use the same kind of formula because like when one kid shook hands with someone else you only counted it once. You count a chocolate and vanilla cone and a vanilla and chocolate cone only once also. It is different because you can have a chocolate and chocolate cone but you cannot talk to yourself on the phone, so I can use part of the formula above

$$x(x-1) \qquad x = \# \text{ of flavors}$$

The answer would be 40 (39) = 1560 different flavors. You can use this formula for any number of flavors. Just substitute the number of flavor for x and you will get the number of different combinations.

Precise math language is used throughout.

The student relates this task to another with which she had experience.

The student achieves a correct solution.

The student explains the similarities and differences along with the reasoning used.

The student generalizes the formula used to work with any number of flavors.