Teacher Instructions: How Many Nickels?

Grade Level: 6 - 8

Task: How Many Nickels?

Standard: Geometry, Spatial Sense, and Measurement

The PTA is thinking of holding a Fun Fair with lots of different activities. One activity would be to guess the number of nickels needed to stack from floor to ceiling in our classroom. We need to help them out by giving them the most accurate answer that we can.

Don't forget to share with the PTA exactly how you figured out the answer so they will be confident in giving the prize to the person who has the closest guess during the fair.

Context – From the Task Author: Students had previously done an activity using a ruler to review the conversion of improper and mixed fractions. I was surprised that they really couldn't find fourths, eighths and sixteenths consistently on the ruler. I realized they needed more work with measuring. I thought this task would get them to use a ruler again in a different context.

What the task accomplishes…

- This task has some computation imbedded in the solution, but it also requires the application of important concepts and skills involved with measurement and margins of error.
- The students need to be accurate in measuring the height of the ceiling but they also have to make a decision as to whether to measure the thickness of one nickel or a stack of nickels.

What students will do…

- Most students went immediately to finding the height of the ceiling; most did this quite quickly and inaccurately.
- Since I had given them meter sticks that had inches and centimeters, they had to regroup after they realized it was not a yardstick. Most had to re-measure the height of the room and did so with more concern for accuracy.
- They then needed to think about the nickel. Some students tried to measure one nickel. Other students found the number of nickels in an inch.
- Most students used inches and seemed uncomfortable measuring with centimeters.

Time Required: One, 45-minute math period

Interdisciplinary Links: This task could link with other tasks involving money and measurement. It could also relate to other fundraising activities.

Teaching Tips…

- Be sure to have plenty of coins available for students to use.
- I found I had to ask some clarifying questions as they worked to get them to measure slowly and more carefully.
- I had to ask one group using the meter stick as a yardstick how many inches were in a yard. As they looked at the meter stick, they realized their error.

- I also asked one group how they might verify their strategy (something I always ask). They decided to make a chart of inches to the number of nickels using the same data used in their first method. Later, that led to an interesting discussion about whether using the same data in different ways verified their strategy or their arithmetic.
- Only one group thought of using centimeters.
- Some students did not see how you could get the same (or similar) answers using very different units of measure. This should lead to an interesting class discussion on the variance of measurement.
- I also asked the students to erase any marks they made on the walls. Maybe chalk would be better.
- In some classes I asked the students to estimate the number of nickels first. I often forget to take advantage of that estimation opportunity.

Suggested Materials: I suggest using lots of coins for each student (or group of students) and meter sticks (that have inches also - so students have a choice and have an opportunity to verify their strategy).

Possible Solution…

- ✓ It depends on the height of the ceilings in your classrooms.
- ✓ Consider measuring the thickness of one nickel (1/16th of an inch) or an inch of nickels, and comparing that to the height of the ceiling as a strategy for an expert or practitioner.
- ✓ The larger the stack of nickels the student's use will decrease the margin of error embedded in this problem.
- ✓ I would like them to know that the margin of error is greatly increased over a strategy that measures a stack of nickels.

Benchmark Descriptors:

- ❑ The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- ❑ The descriptors are generalizations of what student work could look like.
- ❑ It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- ❑ It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Novice

- ✓ A novice might measure the ceiling in centimeters and the thickness of the nickel in inches and mix the two systems.
- ✓ A novice may not be able to measure the ceiling within a reasonable range.
- ✓ This student will use little or no accurate math language and is not likely to attempt a math representation.

Apprentice

- ✓ An apprentice may try to compare the thickness of one nickel to the height of the ceiling, thereby multiplying their error to the point of not being within an acceptable range.

- ✓ An apprentice might also measure accurately, but not be able to do the mathematics correctly.
- ✓ The apprentice will use some math language, and a math representation may be attempted.

Practitioner

- ✓ A practitioner will measure accurately and be able to perform all calculations.
- ✓ A practitioner also measures a stack of nickels, thereby limiting their margin of error.
- ✓ The practitioner will use accurate and appropriate math language and will probably use math representations to communicate.

Expert

- ✓ An expert will do all that the practitioner did, but might also verify their solution by solving the problem using centimeters and then solving it again in inches.
- ✓ The expert will use precise math language and will use accurate and appropriate math representations.
- ✓ The expert may also make mathematically relevant connections or observations.

APS Mathematical Standards…

- ❖ **The math standards stated for this task are aligned to the APS Draft Standards 2000.**

Strand - Geometry, Spatial Sense, and Measurement:

Students will demonstrate an understanding of concepts, properties, and relationships of geometry and measurement through experiences with meaningful mathematical problems, while focusing on identifying, describing, classifying, visualizing, comparing, estimating, and measuring various aspects of shapes and sizes.

Benchmark (6 – 8): The student will understand the relationships between 2- and 3-dimensional shapes and identify, build and transform shapes. The student will use inductive and deductive arguments to solve problems. The student will use metric and customary measurement systems and select the appropriate measurement unit for a given situation.

Performance Standards:
Sixth Grade:

- **Select and apply** appropriate formulas to solve problems.
- **Measure** objects using customary and metric units for length, volume, mass, and area.
- **Explain** both customary and metric units of measurement.
- **Convert** from one unit to another accurately within the same system.

Seventh Grade:

- **Select and apply** appropriate formulas to solve problems.
- **Use** appropriate standard units for estimating measurements.
- **Find** length, area, volume, and angle measures to appropriate levels of precision, selecting appropriate techniques and tools.

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:

Sixth Grade:
- **Select** an appropriate operation to solve situational story problems.
- **Select and use** the appropriate number form (fraction, decimal, or percent) in a variety of situations, including measurement in U.S. and metric systems.
- **Develop and test** strategies for adding and subtracting fractions with like and unlike denominators.
- **Develop and test** strategies for multiplying and dividing fractions.
- **Develop and test** strategies for adding and subtracting decimals.
- **Develop and test** strategies for multiplying and dividing decimals.
- **Estimate and solve** problems involving fractions & decimals, and justify the reasonableness of the solution.
- **Use** the appropriate estimation strategy for a variety of situations.
- **Determine** when an exact answer is necessary or when an estimate is appropriate.

Seventh Grade:
- **Translate** problem-solving strategies into efficient computation using appropriate mathematical terminology.
- **Develop** more than one strategy to solve real-life problem situations involving ratios, proportions, and percents.
- **Estimate and solve** problems involving proportions and **justify** the reasonableness of the solution.
- **Create and write** story problems involving proportions.

Eighth Grade:
- **Select** the appropriate representations to describe thought provoking real-life situations.
- **Develop and evaluate** arguments involving real numbers, their patterns and operations.
- **Develop and use** strategies to estimate the results of rational-number computations and **judge** the reasonableness of the results.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

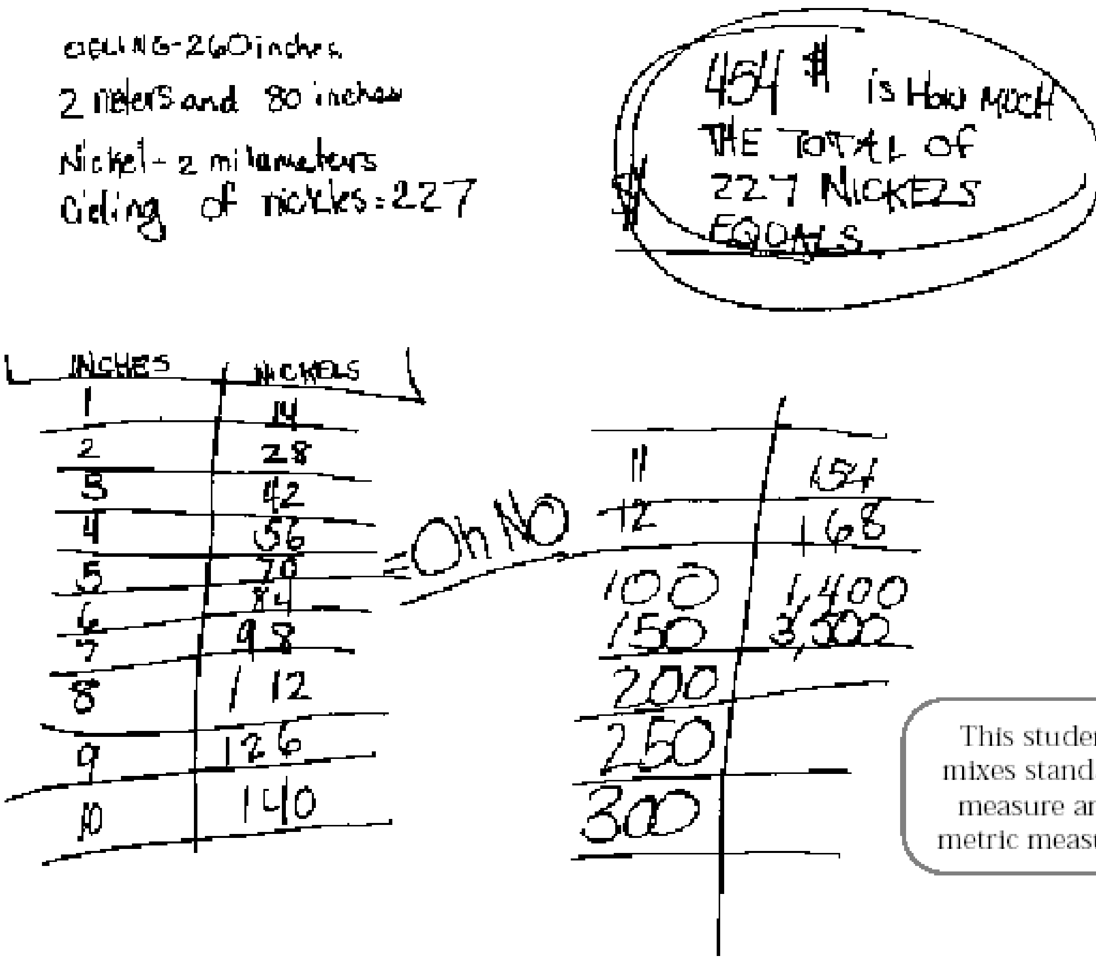
Performance Standards:

Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies,** and **translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

Novice

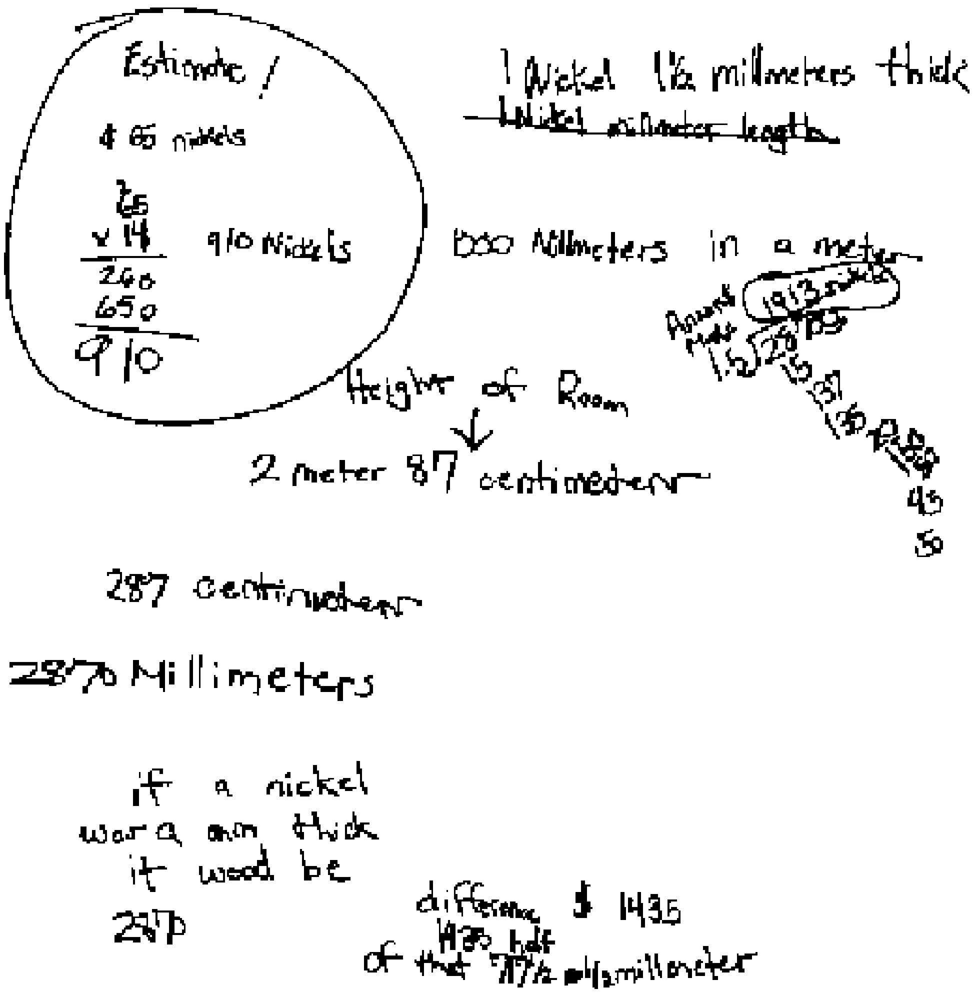


This student mixes standard measure and metric measure.

There is no evidence for the student's solution of 227 nickels and the amount $454.00 does not make sense. The error is in placing the dollars sign.

The student – now deciding to use inches – gets confused as to where the end of their chart should be.

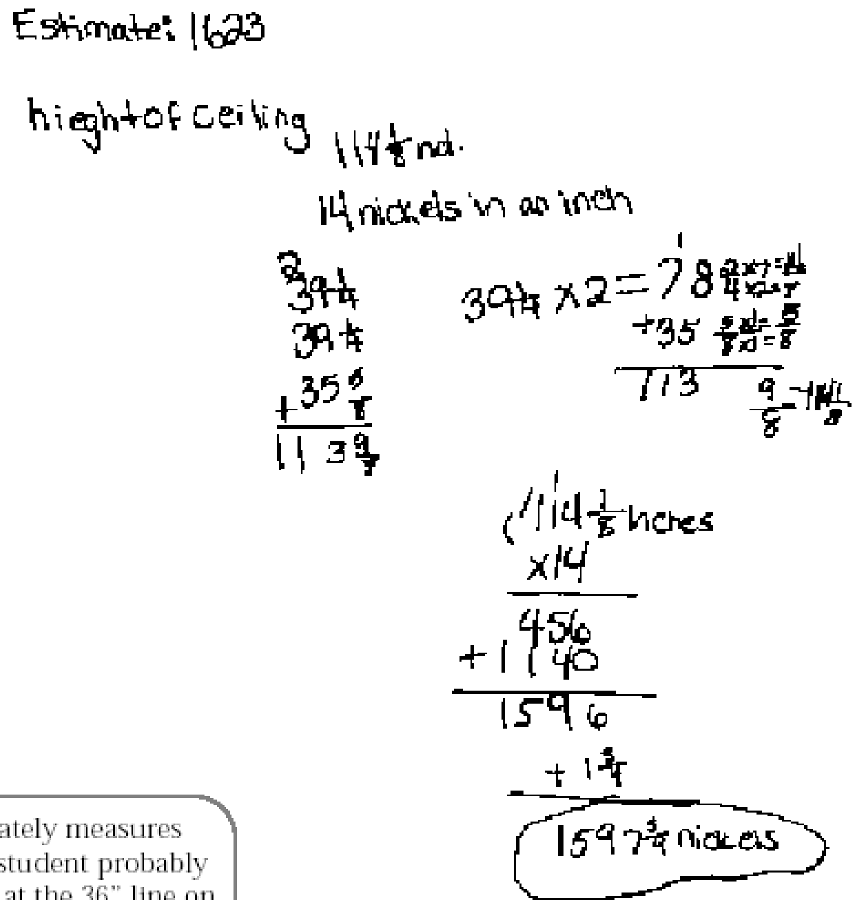
Apprentice



This student tries to compare the height of the ceiling to the thickness of one nickel. This strategy will most likely cause a bigger margin of error than comparing the ceiling height to a stack of nickels.

Interestingly enough, one might think that taking an average of the number of nickels at 1 mm thick and 2 mm thick should give you the number of nickels at 1 1/2 mm.. However, the strategy of averaging does not work. You cannot use an average when the items you are averaging are not the same.

Practitioner



This student accurately measures using fractions. The student probably should have stopped at the 36" line on the meter stick. However, at least they knew it was more than 39" long.

I liked the student's practical use of the distributive property in their multiplication of fractions: 39 1/4 x 2 = [39(2)] + [1/4 x 2]. Also, 114 1/8 x 14 = (114 x 14) + (1/8 x 14).

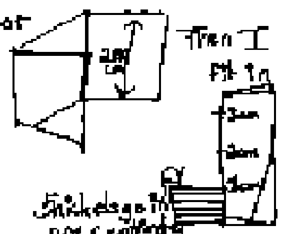
Not much explanation, but you can follow the work fairly easily.

Does the student think that doing the computation two ways is the same as verifying the solution or strategy?

Expert

First I measured the distance from the ceiling to the floor

Then I measured how many nickels fit in one centimeter



Then I multiplied 288 x 5   288 (cm) x 5 (nickels) = 1,440   I got 1,440 nickels

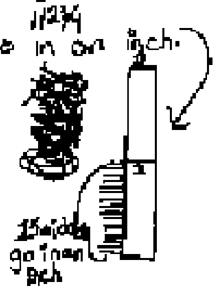
The student shows good explanation of data. The computation is well-labeled.

To check that answer, I measured the distance from the ceiling to the floor in inches

Then I figured out how many nickels would go in an inch.

Then I multiplied 15 x 112

112 x 15 = 560 + 1120 = 1680



Then I realized I got a 220 nickel difference!

Then I re measured step #2, discovered that only 13 nickels go in an inch and redid step #3 And got only a 16 nickel difference.

112 x 13 = 336 + 1120 = 1456

Nice verification using a different measuring system.

The student decides the difference between the two solutions is not acceptable. He or she rechecked data and found an error, making the difference acceptable.