Teacher Instructions: Molly Mathematician's Locker Combination Predicament

Grade Level: 6 - 8

Task: Molly Mathematician's Locker Combination Predicament

Standard: Number Sense and Operations

Molly Mathematician can't remember her gym locker combination. The Physical Education teacher won't provide her with the combination, but did give her the following clues about the combination:

The 3 numbers in the combination are between the numbers 15 and 30.

The sum of the numbers is 70.

When you square each number in her locker combination the digits 1 – 9 will appear only once in the digits of the 3 square numbers.

What is the combination to Molly's locker?
Show all of your work and tell how you achieved a solution by labeling all that you do.

Context – From the Task Author: This task was given to students after studying square numbers and square roots. I wanted to see if they could apply this skill to a problem-solving situation.

What the task accomplishes…

- Not only does this task require students to apply concepts and skills in regard to square roots and square numbers; it also provided opportunity to apply other number sense concepts.
- Since the sum of 3 numbers is 70, then there is a limit on what numbers can be used in the task. For instance, the combination cannot be 3 odd numbers because odd + odd + odd = odd. Nor can it be odd + odd + even. These might be fun rules to explore after solving the task if students don't apply these strategies on their own.

What students will do…

- Most students will begin by listing all possible numbers (15-30) and then squaring them.
- Many students then eliminate all that have O's in the squares, or ones that have more than one of the same digit (such as 15^2 or 225).
- The students will either guess and check, or use some other strategy for finding the possible combinations.
- Students realize that one of the numbers in the combination must be either 19 or 29 since they are the only numbers when squared that contain a 1.

Time Required: 45 minutes

Interdisciplinary Links: This task can be adapted to address many topics. For instance, instead of having this solution be that to a locker combination, it could be 3 numbers that represent ages, ingredients, sport scores, etc.

Teaching Tips…

- Students need to be familiar with square numbers to be successful with this task.

- I started by giving students tiles and asking them explore which numbers of tiles create squares. After listing all of the square numbers, to practice making mathematical connections, students identified patterns and relationships in the list of square numbers and their square roots.
- Several students even came up with formulas for finding square numbers. One example was that of a student who claimed to know how to find the next consecutive square number: Take the square root of the square number you know, double it, add it to the square number you know, then add 1 and you have the next consecutive square number.

  Example: to find the next square number after 100...
  - ✓ Take the square root of 100 = 10 x 2 = 20 + 100 = 120 + 1 = 121 next square number!

Suggested Materials: Calculators

Possible Solution…

✓ There is only one correct solution that meets all requirements of the task: **19-23-28**

$19^2 = 361$

$23^2 = 529$

$28^2 = 784$

19 + 23 + 28 = 70

Benchmark Descriptors:

- ❑ The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- ❑ The descriptors are generalizations of what student work could look like.
- ❑ It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- ❑ It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Novice

- ✓ The novice will not be able to start the task or will only be able to focus on finding 3 numbers whose sum is 70 without regard for other parameters in the task.
- ✓ Little or no math language or representations will be used to communicate.

Apprentice

- ✓ The apprentice will attempt to solve the task, but may arrive at an incorrect solution for one of several reasons.
- ✓ The apprentice may not check to make sure that when the digits of the combination are squared that the digits 1 - 9 are represented, or may use combinations that repeat digits.
- ✓ Some math language will be used to communicate and some math representations may be used to organize work.

Practitioner

- ✓ The practitioner will find a correct solution that addresses all parameters of the task.
- ✓ The practitioner will use accurate and appropriate math language and will have a complete and labeled mathematical representation.

Expert

- ✓ The expert will find a correct solution that addresses all parameters of the task.
- ✓ The expert will clearly explain her/his approach and reasoning.
- ✓ The expert will make mathematically relevant observations about the solution, such as commenting on the characteristics of the numbers in the combination, or by verifying the solution.

APS Mathematical Standards…

- ❖ **The math standards stated for this task are aligned to the APS Draft Standards 2000.**

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:

Fifth Grade:

- **Use** a variety of strategies, including calculators, and geometric models, to find factors, multiples, primes, even/odd numbers, and square numbers, and to explain number composition.
- **Develop, compare and select** a strategy that is efficient and accurate when solving addition and subtraction problems.
- **Develop, compare and select** a strategy that is efficient and accurate when solving multiplication and division problems.

Sixth Grade:

- **Select** an appropriate operation to solve situational story problems.
- **Recognize and use** prime and composite numbers.

Seventh Grade:

- **Translate** problem-solving strategies into efficient computation using appropriate mathematical terminology.
- **Explain and model** the value of exponents and square roots.

Eighth Grade:

- **Select** the appropriate representations to describe thought provoking real-life situations.
- **Manipulate** all real numbers, their properties, and operations.

- **Develop and evaluate** arguments involving real numbers, their patterns and operations.
- **Develop and use** strategies to estimate the results of rational-number computations and judge the reasonableness of the results.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

Performance Standards:

Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies,** and **translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

Novice

It is unclear why some of the numbers are eliminated.

1,2,3,4,5,6,7,8,9

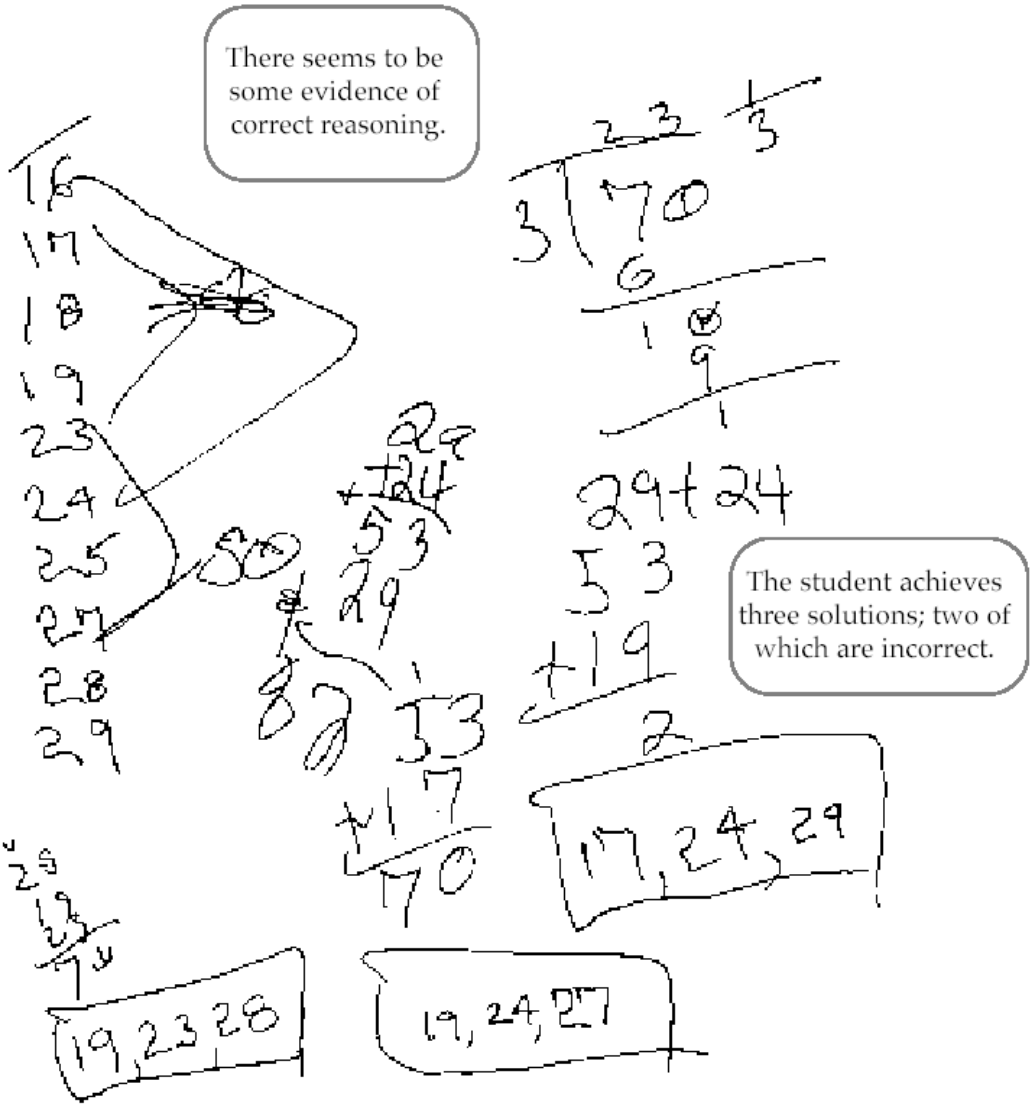
nine is in every one there are two twenty in each.

16 256
17 289
18 324
19 361
20 400
21 441
22 484
23 529
24 576
25 625
26 676
27 729
28 784
29 841

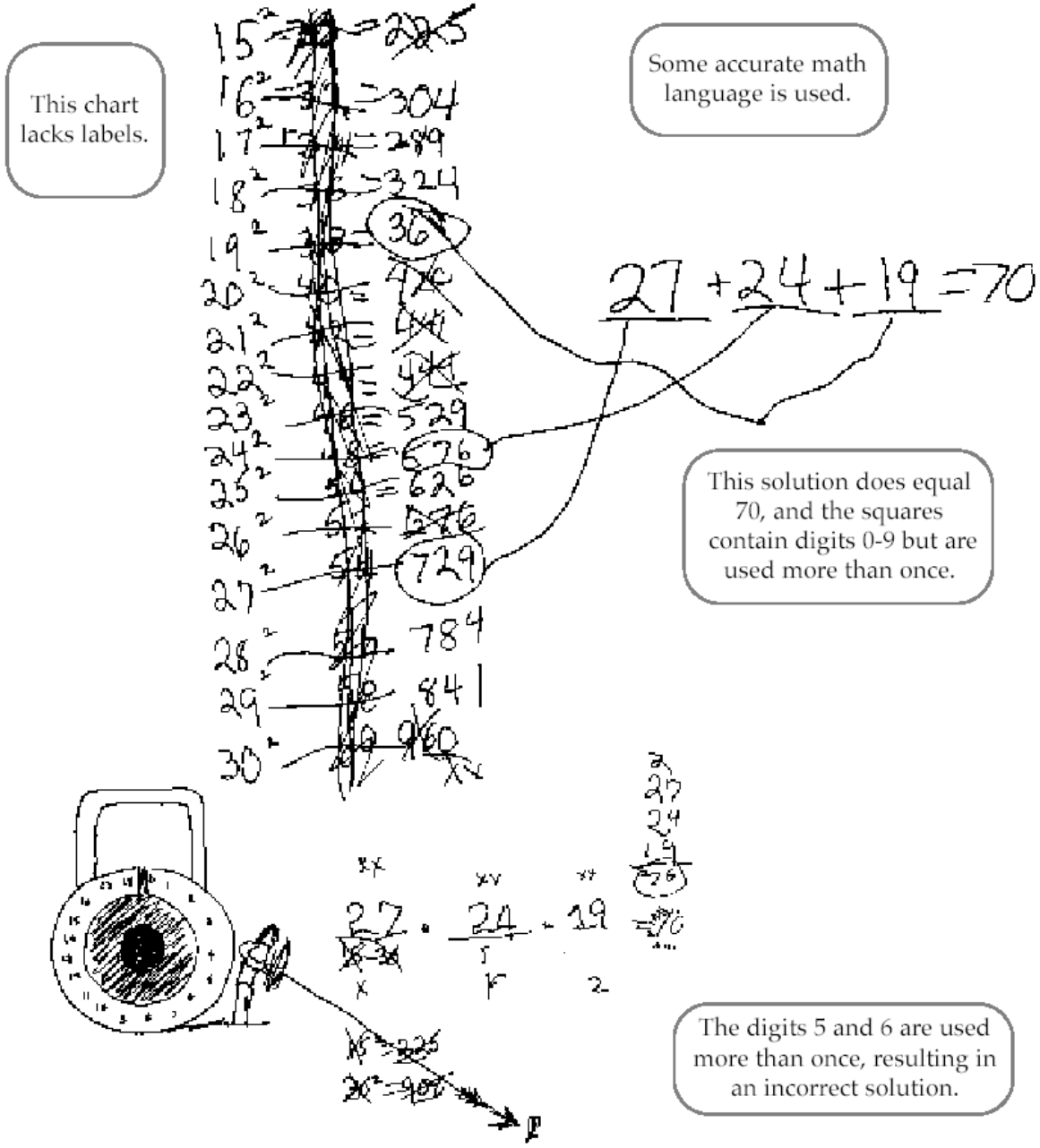
It is unclear what the student means by this.

The student does not use this information to solve the task.

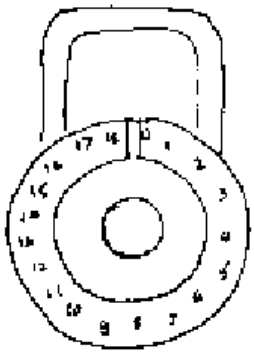
Novice(cont.)

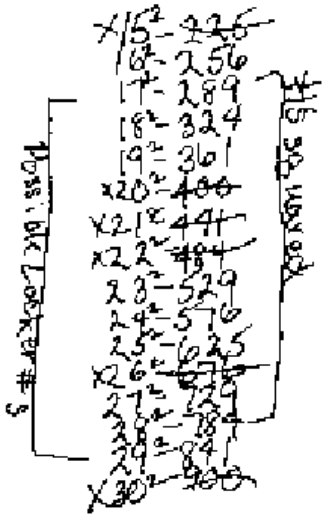

There seems to be some evidence of correct reasoning.
The student achieves three solutions; two of which are incorrect.
16
17
18
19
23
24
25
27
28
29
50
29+24
53
+19
17,24,29
19,23,28
19,24,27

Apprentice


This chart lacks labels.
Some accurate math language is used.
27 + 24 + 19 = 70
This solution does equal 70, and the squares contain digits 0-9 but are used more than once.
The digits 5 and 6 are used more than once, resulting in an incorrect solution.

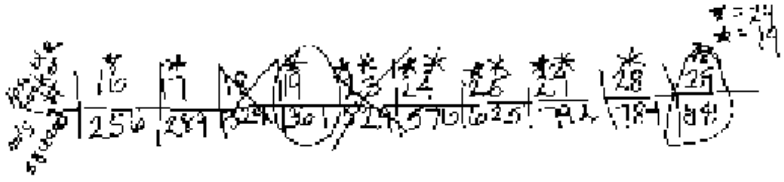
Practitioner

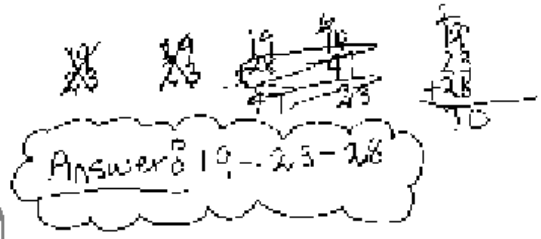




The student uses accurate exponential notation and math language.

The student labels her work.





The student makes an observation that ties to her approach and reasoning.

The student's work is organized in this labeled chart.

One observation I made was there were only two numbers (19 and 23) that had one's in their squared number and since there had to be a 1, 2, 3, 4, 5, 6, 7, 8, 9, in all the squared numbers put together, either 19 or 23 had to be part of the combination.

A correct answer is achieved.

Expert

Molly's Locker Combination Problem

Precise math language is used throughout.

To find the combination of Molly's locker I first made a chart with the numbers 15 through 30 in one column. All of the numbers 15 through 30 were squared in another column, and I wrote if each squared number has any numbers between one and nine without them appearing twice in another column. Here is the formula I used to square the numbers: N x N = S where

N=Number between 15 and 30

S=Number between 15 and 30 squared

Next I started taking random numbers that did have digits 1 through 9 appear and putting them together trying to find a match that had all numbers 1 to 9. On my second attempt, I found three numbers that worked, they were 361, 784, and 529. 361 being 19 squared, 784 being 28 squared, and 529 being 23 squared. The numbers 19, 28, and 23 added up equaled 70. Now I had the combination, it was 19, 28, 23.

The student explains his/her approach and reasoning.

A pattern I found was every time you add a number to a squared number to receive another one you add 2 to the number you add with the squared number every time. For example 225 (a squared number) plus 31 equals 256 (another squared number). 256 plus 33 equals 289 (another squared number). 289 plus 35 equals 324 (another squared number). Every time I added 2 to the number that was being added to the squared number.

Number Between 15 and 30	Number Squared	Have any digits 1 through 9 without being repeated
15	225	no
16	256	yes
17	289	yes
18	324	yes
19	361	yes
20	400	no
21	441	no
22	484	no
23	529	yes
24	576	yes
25	625	yes
26	676	no
27	729	yes
28	784	yes
29	841	yes
30	900	no

The student uses an accurate and appropriate chart in which to organize data.

The student makes a mathematically relevant observation.