Teacher Instructions: Lenny the Lazy Lawn Mower

Grade Level: 6 - 8

Task: Lenny the Lazy Lawn Mower

Standard: Geometry, Spatial Sense, and Measurement

Lazy Lenny mows lawns in the summer for extra money. He wondered how he could mow a lawn with the least amount of walking. So one week, on a 50' X 50' lawn, he mowed the grass in parallel rows (see figure 1). The next week he mowed the same lawn in diagonal rows (see figure 2). The week following, he mowed the lawn in a spiral pattern (see figure 3). His mower cuts a strip about 26" wide.

Which mowing method required the least amount of walking?
Defend your solution mathematically.

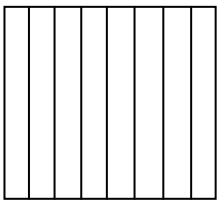


Figure 1

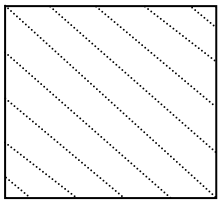


Figure 2

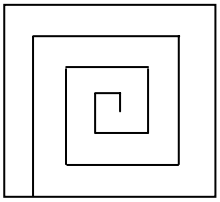


Figure 3

Context – From the Task Author: Lawn mowing is a popular summer job for students of this age group. This task allows students to see how mathematics can be connected to the real world.

What the task accomplishes…

- This task challenges students' spatial sense and allows students to see the relationship between geometry and measurement.
- It also provides a meaningful context in which to apply the Pythagorean Theorem.

What students will do…

- Students will use different approaches to investigate each of the three figures.
- Most will create a diagram or physical model for completing the task.
- All will require the use of calculators.

Time Required: Students spent 60 minutes investigating the solution, and 30 – 45 minutes documenting their work in a final write-up.

Interdisciplinary Links: This task links well to investigations and discussions about finding summer work. Students learn to apply mathematics in their daily lives to save themselves work! The task could also tie to fine arts, or the study of maps.

Teaching Tips…

- Have a variety of materials available to students since the task requires a variety of strategies.
- You may also require that students verify their work by using a second method.

Suggested Materials: Graph paper, colored pencils, scissors, calculators, protractors, and rulers.

Possible Solution…
Figure 1: 1200 feet
Figure 2: 1800 feet
Figure 3: 1150 feet

- ✓ Figure 3 requires the least walking distance.

Benchmark Descriptors:

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- The descriptors are generalizations of what student work could look like.
- It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Novice

- ✓ The novice will not have an approach that will work for finding the distance walked for any of the three figures.
- ✓ The novice will demonstrate little or no mathematical reasoning and will use little or no math language to communicate.

Apprentice

- ✓ The apprentice may show a reasonable determination of the distance walked in figure 1, but may be unable to find an approach for figures 2 and 3.
- ✓ The apprentice may or may not take into account the width of the lawn mower.
- ✓ Some evidence of mathematical reasoning can be inferred, but and incomplete explanation is given.

Practitioner

- ✓ The practitioner will find a correct answer by comparing the distance walked in all 3 figures, even if calculations are estimates and are not exact.
- ✓ These students may utilize information such as the width the lawn mower mows and use effective reasoning.
- ✓ The practitioner's solution is clear, and contains accurate and appropriate mathematical representations.

Expert

- ✓ The expert will find a correct answer by comparing the distance walked in all 3 figures, with exact calculations of the distances.
- ✓ The student will justify the use of the Pythagorean Theorem to solve for figure 2.
- ✓ The student will verify his/her solution.
- ✓ The expert will use clear, precise mathematical language and representations.

APS Mathematical Standards…

❖ **The math standards stated for this task are aligned to the APS Draft Standards 2000.**

Strand - Geometry, Spatial Sense, and Measurement:

Students will demonstrate an understanding of concepts, properties, and relationships of geometry and measurement through experiences with meaningful mathematical problems, while focusing on identifying, describing, classifying, visualizing, comparing, estimating, and measuring various aspects of shapes and sizes.

Benchmark (6 – 8): The student will understand the relationships between 2- and 3-dimensional shapes and identify, build and transform shapes. The student will use inductive and deductive arguments to solve problems. The student will use metric and customary measurement systems and select the appropriate measurement unit for a given situation.

Performance Standards:

Sixth Grade:

- **Develop and test** strategies for finding perimeters and areas.
- **Select and apply** appropriate formulas to solve problems.
- **Measure** objects using customary and metric units for length, volume, mass, and area.

Seventh Grade:

- **Select and apply** appropriate formulas to solve problems.
- **Use** appropriate standard units for estimating measurements.
- **Find** length, area, volume, and angle measures to appropriate precision, selecting appropriate techniques and tools.

Eighth Grade:

- **Apply** the Pythagorean Theorem in appropriate situations to solve problems involving triangles.

Strand – Patterns, Functions, and Algebraic Concepts:

Students will demonstrate an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (6 – 8): The student will use tables, graphs, and symbolic representations of patterns. The student will understand and use variable and linear equations in algebraic problem solving.

Performance Standards:

Sixth Grade:

- **Predict** sequences and patterns involving varying rates of change.
- **Explain** how expressions are used to model functions and patterns.

Seventh Grade:
- **Identify and use** variable expressions and formulas to solve a variety of real-life situations.

Eighth Grade:
- **Develop** exponential functions to represent real-life situations.
- **Represent, describe, and analyze** numerical patterns and relationships using tables, graphs, words, and standard algebraic notation.

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:
Sixth Grade:
- **Select** an appropriate operation to solve situational story problems.
- **Use** the appropriate estimation strategy for a variety of situations.
- **Determine** when an exact answer is necessary or when an estimate is appropriate.

Seventh Grade:
- **Translate** problem-solving strategies into efficient computation using appropriate mathematical terminology.

Eighth Grade:
- **Select** the appropriate representations to describe thought provoking real-life situations.
- **Develop and evaluate** arguments involving real numbers, their patterns and operations.
- **Develop and use** strategies to estimate the results of rational-number computations and **judge** the reasonableness of the results.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

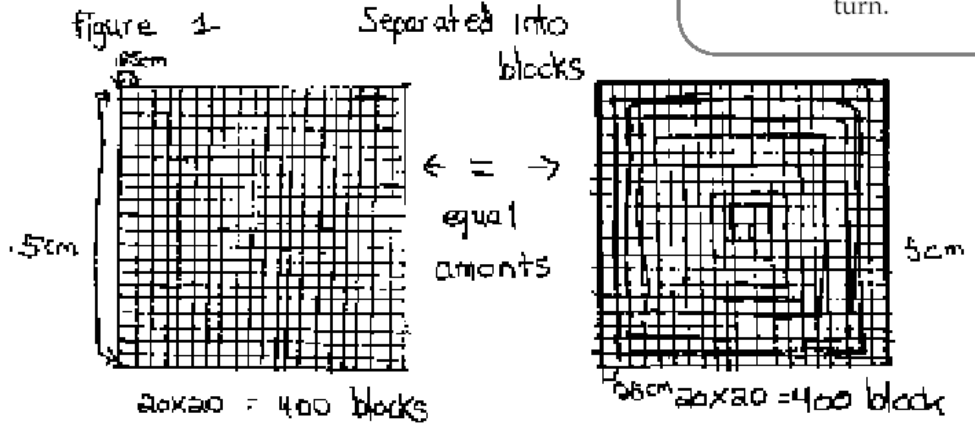
Performance Standards:
Grades Kindergarten through twelve:
- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.

- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies,** and **translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

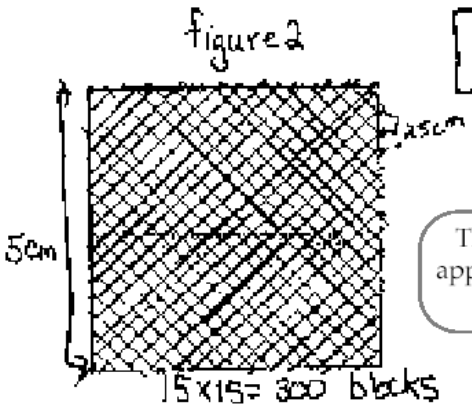
Benchmark Papers

Novice

This solution fails to take into account that each row decreases in length of each turn.



This approach will not lead to an accurate solution since in figure 2 there are many fractional "blocks".



The student attempts to find an approach but it lacks mathematical reasoning.

I would have to say that figure a is the least amount to walk after my way of separating it into smaller blocks.

An incorrect solution is achieved.

Apprentice

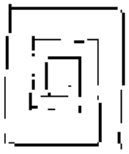


Figure 1

50 x 12' = 600 inches

area of lawn

26)600

-52

80

-78

2

23 stripes x 50 =

1,150 ft.

I took 50 and multiplied it by 12 and got 600 inches. I took 600 inches and divided it by 26 and got 23. I took 23 and multiplied it by 50 and got my answer 1,150 ft.

Accurate math language is used.

The student explains her/his approach & reasoning.

676 + 676 = Figure 2 36.8"

Both solutions are incorrect.

Figure 3

23 x26 = 138 + 460 = 598

26)600 inches

-52

80

-78

2

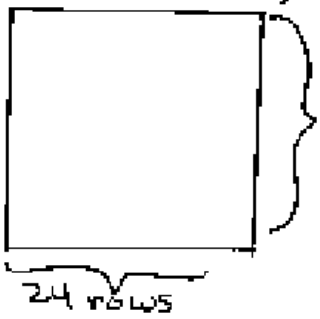
What I did was I figured out these 600 inches and then I divided it 26 and got 23. Then I multiplied 23 by 26 and got 595.

The width of the mower is considered.

An overall conclusion is not determined.

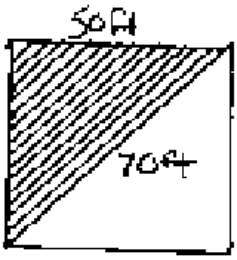
Practitioner

Figure 4 -
If cut into vertical strips it comes to 600 inches on each side. The 600 inches is divided by the 26 inches. This comes out to 23.076923. Because of this small but significant extra, but it is 24 rows wide, 600 inches long. 24 times 600 equals 14,400 inches or 1,200 feet.



The student explains his/her reasoning and approach.

Figure 2 -
In figure 2 it has 37 strips each 26 inches across. I found how long it was by using $A^2+B^2=C^2$ formula. Then multiply the answer by 2. Thus making both sides of the triangle.



inches	length	when x2
26	37	74
52	74	147
78	110	221
104	147	294

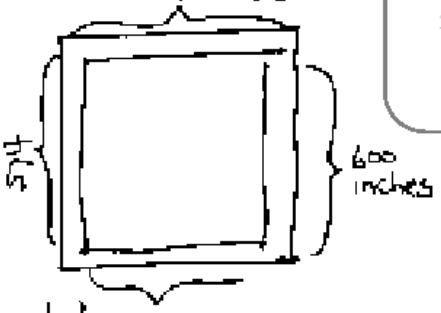
Accurate and appropriate math language is used.

Practitioner (cont.)

Inches	length	when x2
136	197	394
167	229	458
188	265	532
214	302	604
240	339	686
263	376	752
292	412	826
318	449	900
344	496.5	973
370	523.5	1047
396	560	1120
422	596	+1194

9536 inches
or 794 2/3 feet

574 inches



The student makes mathematically relevant observations.

Figure 3-
In figure 3 I found that it had a repeating pattern in it. In this pattern it started at 600 in., but the next side had only 574in. because the first side took up a 50 feet. The third stayed at 574 but the fourth side once again went down by 26 to 548.

Practitioner (cont.)

Row	1	2	3	4	5	6
	600	548	496	444	392	340
	576	522	476	418	366	314
	576	522	476	418	366	314
	548	496	444	392	340	288
	7	8	9	10	11	12
	288	236	184	132	80	28
	262	210	158	106	54	2
	262	210	158	106	54	2
	236	184	132	80	28	

I figured out there would be approximatly 12 rows. When added all together it give a grand totatal of 13,846 in. or 1,153. 8 1/3 feet

Figure 1- 1,200 ft.
2- 794 2/3 ft.
3- 1,153.8 1/3 Ft.

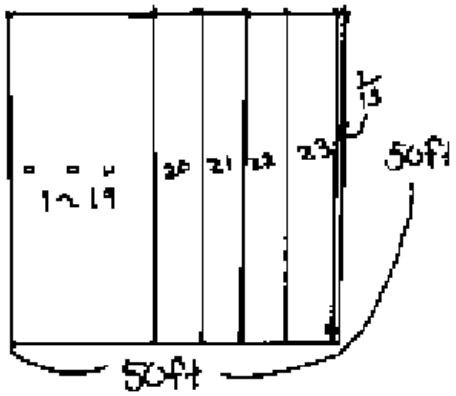
From this data figure 2 is the shortest, figure 3 is second shortest and figure 1 is the longest.

Although figures are not exact, the student has correct work to support her/his solution.

Expert

Fact: Lawn mower cuts 26 inches wide
$26'' = 2\frac{1}{6}$ feet

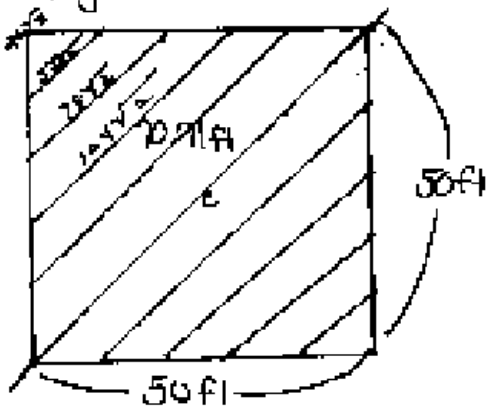
Figure #1



50 feet ÷ $2\frac{1}{6}$ feet = $23\frac{1}{13}$ strips

Walking Distance
= 50 x 24 = 1200
= 1200 ft

Figure #2



Pythagorian theorem

$a^2 + b^2 = c^2$
$50^2 + 50^2 = c^2$
$2500 + 2500 = c^2$
$5000 = c^2$
$\sqrt{5000} = c$
$50\sqrt{2} = c$
70.71 feet = c

How many Strips = $\frac{70.71}{2\frac{1}{6}}$
$= 70.71 \times \frac{6}{13}$
$= 32.64$

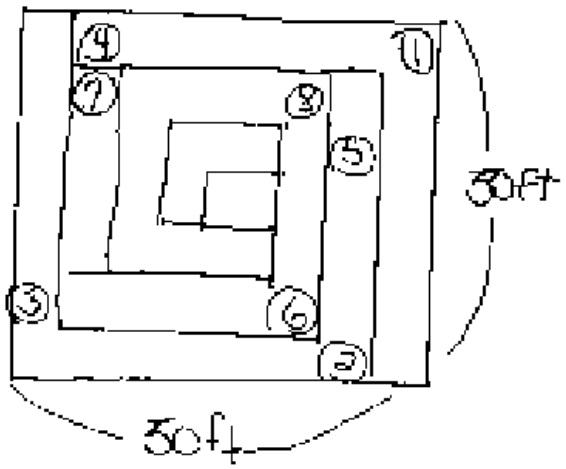
Diagrams assist in clear communication.

The student finds correct solutions to all three parts.

Precise mathematical language is used.

Expert (cont.)

Figure #3



Strip 1 = 50 - $2\frac{1}{6}$

2, 3 = 50 - ($2\frac{1}{6}$ x 2)
4, 5 = 50 - ($2\frac{1}{6}$ x 3)
6, 7 = 50 - ($2\frac{1}{6}$ x 4)
8, 9 = 50 - ($2\frac{1}{6}$ x 5)

42, 43 = 50 - ($2\frac{1}{6}$ x 22)
44, 45 = 50 - ($2\frac{1}{6}$ x 23)

* There is $23\frac{1}{13}$ strips just like Figure #1

Walking Distance

$[(1) + ((2) + (4) + (6)) + \ldots + (44)) \times 2] =$
$47\frac{5}{6} + [(45\frac{3}{6} + \frac{1}{6}) \times \frac{22}{23}] + 2 =$ 1056 $\frac{1}{6}$ Feet

Shortest

An accurate conclusion is drawn.