

Teacher Instructions: John's Field

Grade Level: 6 – 8

Task: John's Field

Standard: Geometry, Spatial Sense, and Measurement

John has decided to fix up an old field for his son's horse. The length of the field is 10 meters less than 4 times its width. First, he fenced in the field at a cost of \$4.80 per meter. The total cost was \$1584. He now needs to buy sweet grass seed to plant in the field. The seed costs \$3.98 per bag and covers 460 square meters.

How much money will John have invested in this field?

Context – From the Task Author: This task was given to students as part of a unit on area and perimeter. The students had access to the formulas for area and perimeter of rectangles and other polygons.

What the task accomplishes...

- Students enjoyed the puzzle nature of this problem, working backwards using the price of the fence to find the perimeter, and then using "guess and test" to find the length and width of the field.
- The task was also challenging for the most advanced students as they applied their algebra skills to finding a solution.

Time Required: Most students completed the task within the 45 minute class period.

Interdisciplinary Links: I use this problem as a tie-in to an interdisciplinary unit on the environment that we as a team do each year. A computer spreadsheet program would be useful for organizing the guesses.

Teaching Tips...

- Students need to have access to formulas for area and perimeter of rectangles.
- They may also need to review strategies for problem solving.
- More advanced students will recognize the possibility for solving for the length and width of the field using algebra.
- **Task Modifications:** This task could be modified by giving the dimensions of the field or by making the field a square.

Suggested Materials: Calculators, graph paper, or computer spreadsheet program

Possible Solution...

- ✓ The total cost of fixing up the field is \$1623.80.
- ✓ Most students will find the perimeter of the field (330 meters) by dividing \$1584 by \$4.80.
- ✓ Then by using either guess or test or an algebraic equation, they will find the dimensions to be 35 meters by 130 meters.
- ✓ The next step would be to find the area (4550 m) and divide by 460 m (the area covered per bag of seed) to decide that 10 bags (rounded) of seed will be necessary.
- ✓ At \$3.98 per bag the cost of seed will be \$39.80 which, added to the cost of fencing, would bring the total to \$1623.80.

Benchmark Descriptors:

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- The descriptors are generalizations of what student work could look like.
- It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Task Specific Rubrics: The intention of the *Performance-Based Mathematics Task Bank* is to provide teachers with materials and experience in working with formative assessments. Teachers are encouraged to modify tasks and/or the task's rubrics to meet the individual needs of their class.

Each monthly task comes with a task specific rubric designed around the tasks grade band K – 2, 3 – 5, or 6 – 8. The task specific rubric may not be appropriate for scoring student work at all of the task's intended grade levels because the scope of performances for each rubric covers 3 grade levels. The rubric may not meet the teacher's specific need for administering the task. Teachers should modify the rubric based on their intention for using the task. A template for creating a task specific rubric has been included in the task bank.

Each teacher should use the task specific rubric, the benchmark descriptors and papers, and the *APS Mathematical Standards* as a guide for assessing their student's performance on a task.

Novice	<ul style="list-style-type: none">✓ The novice will be likely to simply find the sum of the dollar amounts given in the task ($\\$1584 + \\$4.80 + \\$3.98$) and disregard the other information.✓ Little math reasoning will be evident, and little math language used.
Apprentice	<ul style="list-style-type: none">✓ The apprentice will have a strategy that works for part of the task but will not be able to follow through to successful completion.✓ S/he may find the perimeter but then will divide that number by the area covered by each bag of seed instead of finding the length, width, and area of the field.✓ Some math language will be used, and diagrams may be attempted.
Practitioner	<ul style="list-style-type: none">✓ The practitioner will have a strategy to solve all parts of the task, and the answers will be correct.✓ S/he will use accurate and appropriate math language and representation.✓ All work will be included, and it will be easy to follow the student's approach and reasoning.
Expert	<ul style="list-style-type: none">✓ The expert will have a strategy to solve all parts of the task, and the answer will be correct.✓ S/he will probably use an algebraic equation to solve for the dimensions of the field.✓ All math language and representations will be accurate and appropriate.✓ The expert will make a mathematically relevant connection.

APS Mathematical Standards...

❖ The math standards stated for this task are aligned to the APS Draft Standards 2000.

Strand - Geometry, Spatial Sense, and Measurement:

Students will demonstrate an understanding of concepts, properties, and relationships of geometry and measurement through experiences with meaningful mathematical problems, while focusing on identifying, describing, classifying, visualizing, comparing, estimating, and measuring various aspects of shapes and sizes.

Benchmark (6 – 8): The student will understand the relationships between 2- and 3-dimensional shapes and identifies, builds and transforms shapes. The student will use inductive and deductive arguments to solve problems. The student will use metric and customary measurement systems and select the appropriate measurement unit for a given situation.

Performance Standards:

Fifth Grade:

- **Solves** problems that involve perimeter, diameter, base, height, vertices, perpendicular lines, and angles using geometric models of 2-dimensional shapes.
- **Uses** measures of money and time, customary and metric measures of length, weight, and volume to solve problems and **makes** estimates.

Sixth Grade:

- **Develops** and **tests** strategies for finding perimeters and areas.
- **Translates** strategies into formulas for areas and perimeters using appropriate math symbolism (e.g. square feet = ft^2).
- **Selects** and **applies** appropriate formulas to solve problems.
- **Measures** objects using customary and metric units for length, volume, mass, and area.

Seventh Grade:

- **Selects** and **applies** appropriate formulas to solve problems.
- **Uses** appropriate standard units for estimating measurements.
- **Finds** length, area, volume, and angle measures to appropriate levels of precision selecting appropriate techniques and tools.

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:

Fifth Grade:

- **Uses** fractions and decimals to help solve everyday problems.

Sixth Grade:

- **Selects** an appropriate operation to solve situational story problems.
- **Selects** and **uses** appropriate number form (fractions, decimals, or percents) in a variety of situations, including measurement in U.S. and metric systems.
- **Estimates** and **solves** problems involving decimals, and justifies the reasonableness of the solution.
- **Determines** when an exact answer is necessary or when an estimate is appropriate.

Seventh Grade:

- **Translates** problem-solving strategies into efficient computation using appropriate mathematical terminology.

Eighth Grade:

- **Selects** the appropriate representations to describe thought provoking real-life situations.
- **Manipulates** all real numbers, their properties, and operations.

Strand – Patterns, Functions, and Algebraic Concepts:

The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focuses on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (6 – 8): The student uses tables, graphs, and symbolic representations of patterns. The student understands and uses variables and linear equations in algebraic problem solving.

Performance Standards:

Fifth Grade:

- **Uses** variables and open sentences to express simple, single-step algebraic equations ($2 + n = 5$).
- **Investigates** the concept of balance in equations ($7 + 3 = 3 + x$).

Sixth Grade:

- **Analyzes** the use of variables to represent quantities (area of a rectangle: $A = lw$).
- **Explains** that equations are symbolic representations of relationships, patterns, and functions.
- **Solves** one-step equations using the concept of balance when quantities are added, subtracted, multiplies, or divided to both sides of an equation.

Seventh Grade:

- **Identifies** and **uses** variable expressions and formulas to solve a variety of real-life situations.
- **Develops** and **tests** strategies for solving two-step equations.
- **Translates** hypotheses into formal methods of solving algebraic equations.

Eighth Grade:

- **Identifies** and **models** real-life situations using multiple representations.
- **Solves** equations for specified variables (solve for h if $A = bh/2$).

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

Performance Standards:

Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies** and **connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates** and **explains** the mathematics required for various careers.
- **Recognizes** and **applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies, and translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

Novice

$$\begin{array}{r} \text{Fence} = 1584 \\ \text{More Fence} = 4.80 \\ \text{Grass Seed} = 3.98 \\ \hline 1592.78 \end{array}$$

Then he has to buy
a horse which could
really cost alot of
money.

The student's approach of adding
up the numbers given in the task
is one that will not work.

Little or no math
language is used.

No understanding
is shown.

Apprentice



The fence for the field costs
\$4.80 per meter and John pays
\$1584 total he bought $1584 \div 4.80$
330 meters of fence. So if one
bag is enough for 468 meters
then he only needs 1 bag.
That costs \$3.98 so he spent
 $\$1584 + \3.98 .
\$1587.98!

Work is shown and
some parts are correct.

An incorrect solution
is achieved.

Some math language
is used.

The student had a good start in finding
the perimeter but was unable to
continue successfully.

The picture of a horse in a field is not an
appropriate math representation and
does not help solve the task.

Practitioner

John spent \$1584 for fencing at \$4.80 per meter. The perimeter of the field is $\$1584 \div \$4.80 = 330$ meters. I needed to know the length and width of the field to find area but the problem gave me some clues. The length is 10 meters less than 4 times the width and I already know that $2 \times \text{length} + 2 \times \text{width} = 330$. I used guess and check for my strategy. I started by guessing a width then multiplied that number by 4 and subtracted 10. If those worked with my perimeter formula I knew I guessed right.

	width	length	perimeter
1st guess	10m	30m	80m
2nd guess	20m	70m	180m
3rd guess	30m	110m	280m
4th guess	40m	150m	380m
5th guess	35m	130m	330m

Now that I knew the length and width of the field I could multiply those together to get the area. $35\text{m} \times 130\text{m} = 4550\text{m}^2$. So all that was left to was figure out how many bags of grass seed was needed to cover the field and then add all the costs together.

$$4550 \div 460 = 9.891304 \text{ or } 10 \text{ bags.}$$

$$10 \times \$3.98 = \$39.80$$

$$\$39.80 + \$1584 = \$1623.80$$

So, all together John spent \$1623.80 to fix up the field for his son's horse.

Representation is accurate and appropriate.

Accurate math language is used throughout.

The approach is explained.

Guess and check is a strategy that works.

All work is shown.

Expert

$$1584 \div 4.8 = 330$$

the perimeter is 330m

$$x = \text{width}$$

$$4x - 10 = \text{length}$$

$$2(x) + 2(4x - 10) = 330$$

$$2x + 8x - 20 = 330$$

$$10x = 350$$

$$x = 35$$

The width is 35m, and the length is 130m

$$35 \times 130 = 4550$$

$$\text{Area} = 4550 \text{m}^2$$

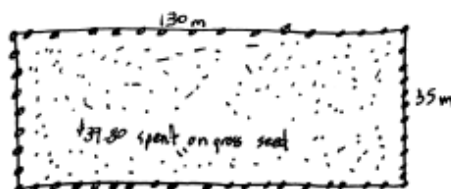
$$4550 \div 460 = 9.891\bar{1}$$

He will need to buy 10 bags of grass seed

$$\$39.80 \text{ spent on grass seed plus } \$1584 \text{ for the fencing} = \$1623.80$$

John has invested \$1623.80 on the field

I took the total cost of the fencing and divided it by the price per meter to find out the perimeter of the field. I then put the perimeter into my equation to find the length and width of the field. Next I multiplied the width by the length to find the area of the field. I took the area and divided it by the area one bag of grass seed to find out how many bags John would need. John would need 9.891 bags. I know you can't buy .981 of a bag so I rounded it up to 10 bags of grass seed. I multiplied the cost of one bag by 10 and added that answer to the total cost for the fencing and got \$1623.80. That is how much money John invested on the field.



\$1584 spent on fencing

Math language is accurate and appropriate throughout.

Representation (model) is appropriate and accurate.

A correct answer is achieved.

The student assigned a variable and successfully solved an equation for the width of the field.

S/he makes a mathematically relevant observation, "I know you can't buy .981 of a bag so I rounded it up to 10 bags..."