

Teacher Instructions: GEARS!

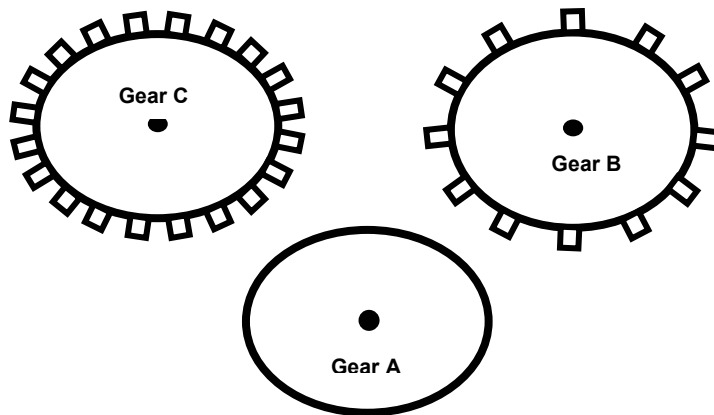
Grade Level: 6 - 8

Task: GEARS!

Standards: Patterns, Functions, and Algebraic Concepts

Look at the set of gears. Gear B has 12 teeth and Gear C has 20 teeth. Find the number of teeth that Gear A has to have so that when it rotates once, Gear B and Gear C have rotated some whole number of times.

Be sure to explain your reasoning clearly. Connect as much mathematics and use as much math language as you can.



Context – From the Task Author: My sixth grade class was just beginning to study gears in science class and had already studied multiples in math class. I wanted to integrate and connect the concepts that were being presented in both classes. The class had been playing with some gears we made out of cardboard and investigating what direction interlocking gears turned and putting different size gears together to see how they turned. As they collected data on their gear ratios many were beginning to see a relationship.

What the task accomplishes...

- This task pulls together some of the investigations the students were conducting.
- I was hoping that most students would recognize the use of multiples in determining gear ratios. We had not studied ratios formally, but had some discussions about ratios and proportions in other problems.
- Some students may be able to connect those ideas also.
- Standard 4 in the *NCTM Standards* (1989) - Mathematical Connections - states that "In grades 5-8 the mathematics curriculum should include the investigation of mathematics connections so that students can apply mathematical thinking and modeling to solve problems that arise in other disciplines such as art, music, psychology, science and business."

What students will do...

- Some students used the cardboard gears while others felt confident to imagine the gears turning.
- Most students used a chart to keep track of the turns the gears made.

Time Required: The task should take about 45 minutes.

Interdisciplinary Links: The study of gears in a science unit.

Teaching Tips...

- Having gears that students can handle would be helpful, especially if this will not be coordinated with a science unit.
- I had templates of gears and cut them out of cardboard and used straight pins as the axle.

Suggested Materials: Cardboard gears

Possible Solution...

- ✓ The least common multiple of 12 and 20 is 60, so gear A has to have 60 teeth.

Benchmark Descriptors:

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels.
- The descriptors are generalizations of what student work could look like.
- It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- It is recommended that teachers create their own task specific rubric by listing the specific math skills that would make up each section of the four performance levels.

Novice	<ul style="list-style-type: none">✓ There is no evidence of a strategy no an answer to the problem.✓ The student shows two multiplication problems that each equal 160, but the numbers do not relate to the problem.✓ There is no explanation of the solution.✓ The student tries to place the gears differently, but that does not seem to help solve the problem.
Apprentice	<ul style="list-style-type: none">✓ The student uses a strategy that is partially useful.✓ Their use of multiples is evidence of mathematical reasoning, but they fail to find the Least Common Multiple and therefore Gear A will turn more than one time.✓ There is some use of mathematical representation and language.
Practitioner	<ul style="list-style-type: none">✓ This student shows a broad understanding of the problem.✓ His/her use of the chart allowed a pattern to be seen.✓ The clear explanation shows that the student visualized the gears moving.✓ The connection to multiples is made and used to find the solution.✓ The student uses effective representation to collect data and find a pattern.
Expert	<ul style="list-style-type: none">✓ This student shows a deep understanding of the problem.✓ S/he not only made the connection to multiples, but also talks about ratios and shows how the ratio was determined.✓ The solution leads directly to a solution and the student employs refined reasoning when they simplified their findings to made ratios.✓ There is also a clear and effective explanation LCM using precise and appropriate mathematical language (multiples, LCM, ratio) and the chart helps communicate the pattern that lead to the solution.

APS Mathematical Standards...

❖ The math standards stated for this task are aligned to the APS Draft Standards 2000.

Strand – Number Sense and Operations:

Students will demonstrate number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student will understand problems involving fractions, decimals, and percents and develop, analyze, and explain a variety of algorithms and methods to solve problems.

Performance Standards:

Fifth Grade:

- **Uses** a variety of strategies, including calculators and geometric models, to find factors, multiples, primes, even/odd numbers, and square numbers, and to explain number composition.

Sixth Grade:

- **Finds** Greatest Common Factor (GCF) and Least Common Multiple (LCM) using a variety of strategies, including prime factorization.
- **Determines** when an exact answer is necessary or when an estimate is appropriate (e.g., medicine dosage Vs number of people at a concert).

Seventh Grade:

- **Translates** problem-solving strategies into efficient computation using appropriate mathematical terminology.
- **Explains** the relationship that can be expressed as ratios of part-to-whole (e.g., 5 red apples out of a total of 8 apples, expressed as $\frac{5}{8}$).
- **Explains** the relationship that can be expressed as part-to-part (e.g., 5 red apples, 3 green apples, expressed as $\frac{5}{3}$).
- **Explains** relationships that can be expressed as proportions or percents (e.g., $\frac{1}{2} = 50\%$).

Eighth Grade:

- **Selects** the appropriate representations to describe thought provoking real-life situations.
- **Manipulates** all real numbers, their properties, and operations.
- **Develops** and **evaluates** arguments involving real numbers, their patterns and operations.
- **Develops** and **uses** strategies to estimate the results of rational-number computations and judge the reasonableness of the results.

Strand - Geometry, Spatial Sense, and Measurement:

Students will demonstrate an understanding of concepts, properties, and relationships of geometry and measurement through experiences with meaningful mathematical problems, while focusing on identifying, describing, classifying, visualizing, comparing, estimating, and measuring various aspects of shapes and sizes.

Benchmark (6 – 8): The student will understand the relationships between 2- and 3-dimensional shapes and identifies, builds and transforms shapes. The student will use inductive and deductive arguments to solve problems. The student will use metric and customary measurement systems and select the appropriate measurement unit for a given situation.

Performance Standards:

Fourth Grade:

- **Represents** and **solves** real-world problems using geometric models.

Sixth Grade:

- **Explains** the properties of circles.

Strand - Data Analysis, Statistics, and Probability:

The student identifies patterns and special features of data and events of chance through experiences with meaningful mathematical problems that focus on comparing, predicting, representing data, and making decisions to communicate mathematical understanding.

Benchmark (6 – 8): The student designs a data question with 2 variables and collects, represents and analyzes the data. The student uses a variety of graphical representations to display data and understand measures of center and spread. The student makes conjectures and computes simple probability outcomes using a variety of tools.

Performance Standards:

Sixth Grade:

- **Selects** and **develops** appropriate display(s) of data (e.g., T-charts, graphs).
- **Develops** and **evaluates** inferences, predictions, and arguments that are based on data.

Eighth Grade:

- **Interprets** data and **makes conclusions** from data.

Strand – Patterns, Functions, and Algebraic Concepts:

The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focuses on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (6 – 8): The student uses tables, graphs, and symbolic representations of patterns. The student understands and uses variables and linear equations in algebraic problem solving.

Performance Standards:

Fourth Grade:

- **Finds** patterns by organizing data in T-charts and **describes** the growing pattern numerically.

Fifth Grade:

- **Uses** T-charts to represent patterning with functions.
- **Uses** patterns and numerical rules to represent and solve problems.

Sixth Grade:

- **Predicts** sequences and patterns involving varying rates of change (e.g., growth over time).

Seventh Grade:

- **Represents, describes, and analyzes** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.

Eighth Grade:

- **Represents, describes, and analyzes** numerical patterns and relationships using tables, graphs, words, and standard algebraic notation.

Strand - Global Mathematical Processes:

Students will understand and use mathematical process.

Benchmark (K - 12): The student will use problem solving, reasoning and proof, communication, connections, and representation as appropriate in all mathematical experiences.

Performance Standards:

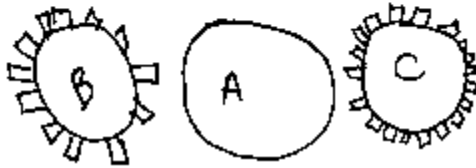
Grades Kindergarten through twelve:

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies for solving given problems.
- **Monitors and reflects** on the process of mathematical problem solving.
- **Makes and investigates** mathematical conjectures and use them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (statement/reasons, proof, informal proof, and algebraic steps).
- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to share from each other in communicating findings.
- **Relates** applications to mathematical language in various modalities.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.
- **Identifies and connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates and explains** the mathematics required for various careers.
- **Recognizes and applies** mathematics in contexts outside the mathematics course.
- **Develops** a repertoire of mathematical representation that can be used purposefully, and appropriately interchangeably (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models).
- **Selects, applies, and translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.

Benchmark Papers

NOVICE

$$16 \times 10 = 160$$
$$8 \times 20 = 160$$



APPRENTICE

It has to be a multiple
of 12 and 20 the
LCM of 20 and 12 is 120
so gear A has to have
teeth equal to a multiple of 120

12	20
12	20
24	40
36	60
48	80
60	100
72	120
84	
96	
108	
120	

multiples
of 120

gear gear gear				
A	B	C	turns	
60	12	20	1)	PRACTITIONER
1	24	40	2)	
	36	60	3)	
	48	80	4)	
	60	100	5)	

I got my answer by making a chart. Then I found out that if I rotated the gear one turn, twelve would have doubled as would twenty because B and C would of turned all the way around. If I turned it again (my third turn) it would increase B by 12, making it 36 and C would of increase by 20 making it forty. On turn four B increases by 12 to forty-eight and C increases to again by twenty equalling to eighty, On turn five B increased by 12 equalling it to be 60 and C increased by twenty equalling one hundred. When I got to turn five I noticed that two multiples were the same— sixty. So that made consider that A had sixty teeth that was the first place that I saw they matched.

You just have to figure out what the GCM is for 12 and 20

EXPERT

12 ~~12, 24, 36, 48~~ LCM 60.
20

* The ratio ~~is~~ For B will be 1:5 the ratio r c will be 1:3 B will go around 5 whole times and C will go around 3 whole times.

* Both ~~of~~ GEARS B and C have to ~~be~~ rotate on even number of times, to do this ~~on~~ GEAR A has to have a number of teeth that is a multiple of 12 and 20. ~~the~~ The LCM between these numbers is 60, the number of

* 12 ~~12, 24, 36, 48~~ LCM 60
20

$12 \times 5 = 60$ (the LCM)
 $20 \times 3 = 60$ (the LCM)

$\frac{12}{60} = \frac{1}{5}$ $\frac{20}{60} = \frac{1}{3}$

These fractions in their simplest forms are 1:5 and 1:3. The ratios

	Multiples by 1	Multiples by 2	Multiples by 3	Multiples by 4	Multiples by 5	
Multiples of 12	12	24	36	48	60	60 is the least common multiple (LCM) between 12 and 20.
Multiples of 20	20	40	60			