Teacher Instructions: Candy Dilemma

Task: Candy Dilemma
APS Mathematics Standard: Number Sense and Operations
Grade Level: 6 - 8

I bought a box of candy for myself last week. However, by the time I got home I had eaten ¼ of the candies. As I was putting the groceries away, I ate ½ of what was left. There are now 6 chocolates left in the box.

How many chocolates were in the box to begin with?

Be sure to show and explain all of your reasoning.

Context – From the Task Author: My sixth grade class was beginning to work with fractions. I knew they had some experience with fractions in fifth grade, but I wanted to assess their flexibility in working with fractions in a problem solving situation. Some times in my class I give an assessment piece and I don't encourage or discourage kids discussing the problem. Usually if it is a problem that is very challenging, most kids will need to discuss the problem with another peer to clarify their own thinking or strategy. I know that most of the time these problems would not have been solved as nicely - if at all - if they had to solve them all on their own. However, I try to make my classroom mirror real situations where problem solving is not usually done in a vacuum. Other times, I want to find a problem that I feel is challenging and yet at a level that can be solved by most of the students without peer collaboration. The Candy Dilemma is a problem I gave to my class to solve on their own without a discussion with peers.

What the task accomplishes…
- ➢ This task allows the teacher to assess their students' ability to problem solve on their own.
- ➢ It is a problem that most kids can at least begin to approach individually.
- ➢ It allows the teacher to assess their students' flexibility to use and solve fractions, and to determine where the students are functioning with their understanding of fractions.

What students will do…
- ✎ Some students began by drawing pictures.
- ✎ Other students used a guess and check strategy.
- ✎ Some tried to work backwards and were successful.

🕐 **Time Required:** One 45-minute period

Interdisciplinary Links: None

Teaching Tips…
- ☆ Some students came up with the incorrect solution of 24 candies fairly quickly. They thought 1/4 + 1/2 = 3/4 so the 6 candies left must be 1/4 of the box. They had not drawn a diagram or really thought about their thinking.
- ☆ I encouraged them to "check" their solution by drawing a diagram - some realized their error when they did check.
- ☆ This is a good problem to ask students to show more than one strategy.

Suggested Materials: Some students may want to use graph paper or fraction bar manipulatives. I also had students who wanted to use a circle graph to solve the problem and needed a compass and protractor.

Possible Solution…
- ✓ The six candies represent 1/2 of 3/4 of a box = 3/8 Therefore, each eighth of the box has 2 candies.
- ✓ This makes 16 candies in a full box.

Benchmark Descriptors:

- The benchmark descriptors and rubric are designed to help the teacher analyze student thinking and understanding at each of the four performance levels (Novice, Apprentice, Practitioner, and Expert).
- The descriptors are generalizations of what student work could look like. It is not possible to anticipate every answer a student can give, so in scoring student work the teacher must use these generalizations to come to their own conclusions as to where a student is performing on the assessment.
- It is recommended that teachers use the task specific rubric given for the assessment to identify the specific math skills that make up each section of the four performance levels for the task.
- Teachers should also review the benchmark papers provided to get a sense of the mathematics that students will use to solve the task.
- If the student does not attempt to solve the task or the work on the problem is completely unrelated to the task, the student's work on the task is considered 'Unscorable' and should not be assigned a performance level of Novice, Apprentice, Practitioner, or Expert.

General Rubric – Reflects the students' performances on the Benchmark Papers

Novice	✓ This student is using inappropriate concepts to solve the problem. ✓ S/he is trying to add 1/2 and 1/4 by finding a common denominator. ✓ The explanation is incomplete as to how s/he found the answer to be 24 candies, except to assume they thought 3/4 of the box of candies were eaten and 1/4 or 6 candies were left. ✓ The strategy of adding 1/4 and 1/2 and thinking that is the amount of candy eaten is an error in mathematical reasoning and a strategy that is not helpful in solving the problem. ✓ There is no use of mathematical representation.
Apprentice	✓ This student's solution is not complete (s/he gives the fraction of the box that is left uneaten) indicating part of the problem is not understood. ✓ The student uses a strategy that is partially useful leading some way toward a solution, but not to the full solution of how many candies are in the box. ✓ There is evidence of mathematical reasoning, but the student did not take the solution to the end. ✓ There is some use of mathematical representation (the circle graph and the rectangular representation of the box of candies) and mathematical terminology (degrees, fractions). ✓ There were students who were able to follow through with the idea of a circle graph to successfully solve the problem. These students I rated as Expert because of their ability to connect central angles and fractions.
Practitioner	✓ This student uses guess and check as a solution. Through guess and check, it is clear the student understands the major concepts necessary for the solution. ✓ There is a clear explanation and the student uses mathematical representation as well as mathematical terminology.
Expert	✓ This student has a deep understanding of the problem including the ability to use beginning algebra concepts (without having taken any formal algebra course) as an efficient and sophisticated strategy leading directly to a solution. ✓ There is a clear and effective explanation detailing how the problem was solved. ✓ The student includes a diagram that shows a possible second strategy to solve the problem. ✓ There is precise use of mathematical terminology and notation (1/2 of 3/4C is 3/8 C; reciprocal; 48/3 = 16).

Task Specific Rubrics: A task specific rubric has been developed for this task at grades K through 8. The rubric reflects the target performance standards listed below for each grade level. This task is not a district assessment; teachers should feel free to modify the rubrics that are given. A template has been given so teachers can modify the rubrics for their classroom use.

There are three key steps to creating a task specific rubric:

1. Identify the target performance standards the teacher intends to measure.
2. Establish student proficiency (practitioner) levels for the student's understanding, strategy and communication of the task in relation to the target performance standards identified.
3. List criteria for the student performance levels of Novice, Apprentice, and Expert in relation to the target performance standards identified.

➢ Teachers can track their individual student's performances on the Standards Matrix provided for the task.

APS Mathematics Content and Performance Standards…
Target Performance Standards 6 - 8:

Grade 6:

- **Translates** hypotheses into formal and fluent fractional and decimal computations using appropriate mathematical terminology and processes.
- **Develops and tests** strategies for adding and subtracting fractions with like and unlike denominators.
- **Develops** and **tests** strategies for multiplying and dividing proper, improper, and mixed fractions.
- **Explains** that equations are symbolic representations of relationships, patterns, and functions.

Task Proficiency: This is a good task for the 6th grader to use the fraction manipulatives to explore the fraction relationships. The student needs to understand what fraction of the candy remains in the box after the candies have been eaten. Students need to understand that the 6 candies remaining represents ½ of ¾, which is 3/8. Good strategies for this task would be working backwards and having the students make the fraction connections or trial and error. If the students are going to use trial and error they should explain why they choose their starting numbers. At the sixth grade level students do not have to be able to translate this problem algebraically.

Teachers should look for the following concepts in the student's performance:

- Can the student use the fraction manipulatives to represent the problem?
- Does the student use trial and error to determine the total amount of candy in the box? What numbers (12, 16, 18, 24, 28) do they use, are they reasonable places to start?
- Does the student understand what the fractions represent in the problem?

Grade 7:

- **Explains** the relationship that can be expressed as ratios of part-to-whole (e.g., 5 red apples out of a total of 8 apples, expressed as 5/8).
- **Explains** the relationship that can be expressed as part-to-part (e.g., 5 red apples, 3 green apples, expressed as 5/3).
- **Translates** hypotheses into formal methods of solving algebraic equations.

Task Proficiency: The 7th grader should begin to explore this problem graphically and translate the picture into a numerical solution. The key concept the student must understand is that the 6 candies that remain in the box represents ½ of ¾, which is 3/8 of the box. The student should be able to reason out (numerically or graphically) that 3/8 = 6, so 1/8 = 2 (6÷3 = 2). Students may also realize that the whole box is 8/8, then see the proportion 3/8 = 6/*x* (3*x* = 48; 48÷3 = 16), and determine that there were 16 pieces of candy in the box.

Teachers should look for the following concepts in the student's performance:

- How does the student approach the problem? If they are having difficulties let the student reason the problem out with fraction manipulatives.

- Can the student then make the connections from the manipulatives to the graphic representation to the numerical equation? Where does the connection break down?
- How fluent is the student with their use of fractions? Does the student understand when they need to add, subtract, multiply & divide?

Grade 8:

- **Shows** flexibility using multiple number representations; **identifies** relationships involving the subsets of the real number system (e.g., order, least to greatest: 1, $\sqrt{2}$, $\sqrt{3}$, 2).
- **Manipulates** all real numbers, their properties, and operations.
- **Develops and evaluates** arguments involving real numbers, their patterns and operations.
- **Develops and tests** strategies for solving multi-step equations.

Task Proficiency: The 8th grader should be able to explore this problem algebraically. They should be able to graphically represent the fractional portions of the candies in the box, realize that the heart of the problem is that the 6 candies that remain in the box represents ½ of ¾, which is 3/8 of the box. The student may be able to lay the entire problem out algebraically or represent part of the problem algebraically.

Step 1: b = box of candy; ¼ b = first eating binge
Step 2: 4/4 –1/4 = ¾ (what is left in the box)
Step 3: ½ x 3/4b = 3/8b
Step 4: 3/8b = 6; b = 16; There were originally 16 pieces of candy in the box.

Teachers should look for the following concepts in the student's performance:

- Does the student's graphic representation help them progress the problem to a numerical equation?
- What strategy did the student use to solve the equation? Did s/he use ratios and proportions or could they translate the information algebraically? Where does the connection break down?
- Does the student understand the fractional relationships in the problem?

Alignment of the APS Mathematics Content and Performance Standards

Strand II – Number Sense and Operations:
The student demonstrates number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmark (6 – 8): The student understands problems involving fractions, decimals, and percents and develops, analyzes, and explains a variety of algorithms and methods to solve problems.

Performance Standards:
Sixth Grade:
Simple Expressions

- **Uses** commutative, associative, identity, zero, and distributive properties when solving problems.
- **Selects** an appropriate operation (i.e., +, -, x, ÷) to solve situational story problems.

Rational Numbers/Fractions

- **Selects and uses** the appropriate number form (e.g., fraction, decimal, or percent) in a variety of situations, including measurement in U.S and metric systems.
- **Explains** the part-whole relationships in division situations (e.g., ½ = 1 ÷ 2).
- **Orders** a mix of fractions, decimals, and percents.
- **Describes** patterns within and among sets of fractions, decimals, and percents (e.g., if 1/8 = .125, 2/8 = .250, what does 7/8=?).
- **Describes** the effects of arithmetic operations on fractions and decimals.
- **Finds** Greatest Common Factor (GCF) and Least Common Multiple (LCM) using a variety of strategies, including prime factorization.
- **Develops and tests** strategies for adding and subtracting fractions with like and unlike denominators.
- **Develops** and **tests** strategies for multiplying and dividing proper, improper, and mixed fractions.
- **Translates** hypotheses into formal and fluent fractional and decimal computations using appropriate mathematical terminology and processes.

- **Estimates and solves** problems involving fractions and **justifies** the reasonableness of the solution.

Seventh Grade:

Proportional Reasoning

- **Explains** the relationship that can be expressed as ratios of part-to-whole (e.g., 5 red apples out of a total of 8 apples, expressed as 5/8).
- **Explains** the relationship that can be expressed as part-to-part (e.g., 5 red apples, 3 green apples, expressed as 5/3).
- **Explains** relationships that can be expressed as proportions or percents (e.g., ½ = 50%).
- **Develops** more than one strategy to solve real-life problem situations involving ratios, proportions, and percents.
- **Estimates and solves** problems involving proportions and **justifies** the reasonableness of the solution.

Eighth Grade:

Rational and Irrational Numbers

- **Categorizes** numbers and sets within structures of the real number system (e.g., Natural numbers < rational numbers < real numbers).
- **Shows** flexibility using multiple number representations; **identifies** relationships involving the subsets of the real number system (e.g., order, least to greatest: 1, $\sqrt{2}$, $\sqrt{3}$, 2).
- **Selects** appropriate mathematical representations to describe thought-provoking real-life situations.
- **Manipulates** all real numbers, their properties, and operations.

Number Theory

- **Develops and evaluates** arguments involving real numbers, their patterns and operations.
- **Develops and uses** strategies to estimate the results of rational-number computations and **judges** the reasonableness of the results.

Strand V – Patterns, Functions, and Algebraic Concepts:

The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmark (6 – 8): The student uses tables, graphs, and symbolic representations of patterns. The student understands and uses variables and linear equations in algebraic problem solving.

Performance Standards:

Sixth Grade:

Patterns

- **Predicts** sequences and patterns involving varying rates of change (e.g., growth over time).

Functions

- **Compares and contrasts** models of continuous functions (e.g., plant growth over time) in real-life applications.

Variables**, Expressions, and Equations**

- **Analyzes** the use of variables to represent quantities (e.g., area of a rectangle: A = *lw*).
- **Explains** how expressions are used to model functions and patterns [e.g., 2, 4, 6, 8 represents $f(x) = 2n$].
- **Explains** that equations are symbolic representations of relationships, patterns, and functions.
- **Solves** one-step equations using the concept of balance when quantities are added, subtracted, or divided to both sides of an equation.

Seventh Grade:

Variable Expressions

- **Represents, describes, and analyzes** numerical patterns and linear relationships using tables, graphs, words, and standard algebraic notation.

Functional Relationships

- **Translates** hypotheses into formal methods of solving algebraic equations.

Eighth Grade:

Functions

- **Represents, describes, and analyzes** numerical patterns and relationships using tables, graphs, words, and standard algebraic notation.

Linear Equations and Inequalities

- **Identifies and models** real-life situations using multiple representations.
- **Simplifies** algebraic expressions including rational expressions.
- **Investigates and applies** the basic mathematical properties (e.g., commutative, associative, distributive, identity, and zero) in a variety of situations.
- **Develops and tests** strategies for solving multi-step equations.

Strand I – Global Mathematical Processes:

The student understands and uses mathematical processes.

Benchmark (K – 12): The student uses problem solving, reasoning and proof, communications, connections, and representations as appropriate in all mathematical experiences.

Performance Standards:

Grades Kindergarten through twelve:

Problem Solving and Reasoning

- **Develops** resourcefulness and perseverance in problem solving in mathematics and other disciplines.
- **Recognizes** when to use previously learned strategies to solve new problems.
- **Develops and uses** strategies (e.g., breaking complex problems into simpler parts) for solving given problems.
- **Monitors, discusses, and reflects** on the process of mathematical problem solving.

Reasoning and Proof

- **Makes and investigates** mathematical conjectures and **uses** them successfully in developing and evaluating mathematical arguments and proofs.
- **Uses** the concept of counterexample to test the legitimacy of an argument.
- **Develops** a logical sequence of arguments leading to a valid conclusion or solution to a problem (e.g., statement/reasons, proof, informal proof, and algebraic steps).

Communication

- **Works** in teams to share ideas, to develop and coordinate group approaches to problems, and to communicate findings.
- **Communicates** mathematical thinking coherently and clearly to others.
- **Analyzes and evaluates** mathematical thinking and strategies of others.

Connections

- **Relates** applications to mathematical language in various modalities.
- **Identifies and connects** functions with real-world applications.
- **Identifies** how seemingly different mathematical situations may be essentially the same (e.g. the intersection of two lines is the same as the solution to a system of linear equations).
- **Investigates and explains** the mathematics required for various careers.
- **Recognizes and applies** mathematics in contexts outside the mathematics course.

Representations

- **Develops** a repertoire of mathematical representation (e.g. pictures, written symbols, oral language, real-world situations, and manipulative models) that can be used purposefully and appropriately interchangeably.
- **Selects, applies, and translates** among mathematical representations to solve problems.
- **Uses** representations to model and interpret physical, social, and mathematical phenomena.
- **Uses** manipulatives, calculators, computers, and other tools as appropriate in order to strengthen mathematical thinking, understanding, and power to build upon foundational concepts.

Benchmark Papers